

Stress calculation from measured strains in the elastic deformation range

by Stefan Keil

Experimental stress analysis using strain gages is carried out in two sequential steps. In the first step the strains created by the stress state under study are measured; in the second step the stress state is calculated from the measured strains. The present paper deals with the second step and is restricted to the elastic deformation range of material. After a general consideration of the relationships between stress and strain in the biaxial stress state, the formulas for the evaluation of strain gage rosette readings are explained. Particular attention is directed to the prevention of errors in the determination of the principal directions. A practical example of stress analysis, consisting of the calculation of principal stresses and their directions from strains measured with strain gage rosettes concludes the paper.

The application of strain gages in monitoring and safety verification on structural members has increased over the past several years. Despite significant advancements in calculational methods - mainly due to the development of the computer - it is still necessary to employ experimental stress analysis. In addition to the cases where experimental strength verification is required for safety reasons, experimental stress analysis is also essential for the determination of stress distributions on complex structural members, or on structures for which calculational techniques are too difficult or involve unacceptable cost.

One of the primary advantages of strain-gage technology is its easy application on structural members in their normal operating environment. In many cases strain gages allow stress analysis under operating conditions of the measuring object. With knowledge of the necessary material laws, the stress condition at the measuring point can be determined from the measured strains. Moreover, the capital investment for instrumentation used in strain-gage measurement is small. These are some of the reasons why strain gages are the preferred technique for experimental stress analysis.

Stress analysis using strain gages consists of two sequential steps: measurement of the strain created by stress in the surface of the specimen, and calculation of stress from the measured strain. This article is concerned primarily with the second step: the calculation of mechanical stress found in material surfaces using the measured strain. This assumes the ability to carry out satisfactory strain measurements using strain gages. Fig. 1 shows typical measurement grids of various strain gages used for stress analysis.

Stress analysis as it is applied in this article is limited to consideration of the elastic deformation range of materials, wherein strain is proportional to stress and is reversible. Upon relaxation of the load in the elastic range, the material returns to its original form. Most analyses using strain gages are limited to the elastic deformation range before the yield point of the mate-

rial is reached. Not covered in the article is stress analysis of elastoplastic deformation, wherein materials under stress exhibit both elastic and plastic (irreversible) deformation. Stress analysis using strain gages in the elastoplastic range is feasible using suitable strain gages and evaluation techniques [1], wherein it is taken into account that stress is no longer proportional to strain.

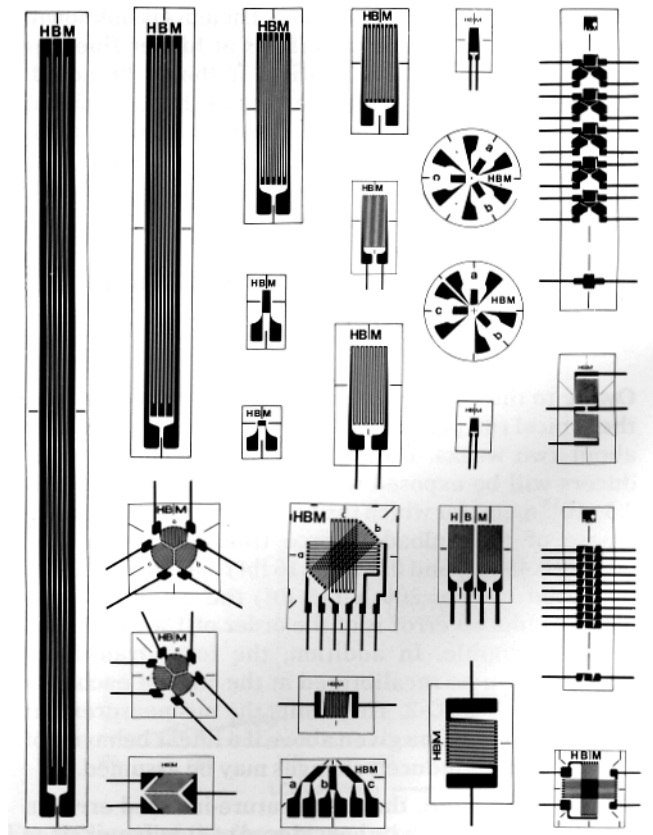


Fig. 1: A selection of the wide variety of strain gages available for experimental stress analysis

Hooke's Law

Stress calculations from measured strains are based on the material laws describing the relationships between stress and strain. In the elastic deformation range of homogeneous materials Hooke's Law is valid. In order to demonstrate this law we consider a cylindrical shaped rod subjected to a tensile force as illustrated in Fig. 2. The unloaded rod, indicated by the dotted line, is of length l_0 diameter d_0 and cross-sectional area A_0 . A force F along the axis of the rod creates a load expressed by the stress σ , i.e. the force per unit of initial cross-sectional area.

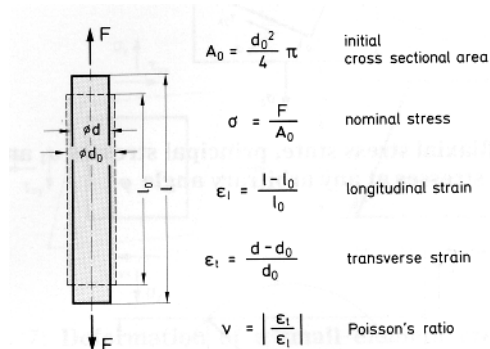


Fig. 2: A rod of circular cross section under tensile load. Longitudinal and lateral deformations with uniaxial stress are shown

The result of this load on the rod is deformation, illustrated by the solid line in Fig. 2. The rod has been stretched by the tensile force F and experiences a longitudinal strain ϵ_l . At the same time, the diameter of the rod has changed. This transverse strain has an inverted sign and is therefore called a lateral contraction. The quotient of the transverse strain ϵ_t and longitudinal strain ϵ_l is called Poisson's Ratio ν . Poisson's Ratio plays a particularly important role in stress analysis using strain gages.

Tension tests performed on cylindrical rods have been used to confirm the linear relationship between stress σ and longitudinal strain ϵ_l . This relationship is known as Hooke's Law. The proportionality factor between stress and strain, which varies from material to material, is called the modulus of elasticity, or Young's Modulus E . Hooke's Law for the uniaxial stress state is

$$\sigma = E \epsilon_l. \quad (1)$$

This form of Hooke's Law is only applicable for the uniaxial stress state, and only when the strain ϵ_l occurs in the axis of the uniaxial stress state. It has to be expected, however, that at the surface of loaded specimens biaxial stress fields occur. Exceptions include uniaxial stress, which is demonstrable in laboratory tests, and triaxial stress on surfaces under pneumatic or hydraulic pressure.

Since Hooke's Law as expressed in equation (1) is only applicable for the uniaxial stress state, it must be expanded for general application. Figure 3 serves to illustrate the necessary considerations. Fig. 3a shows the longitudinal elongation and lateral contraction re-

sulting from a principal stress σ_1 acting on a thin plate. When the plate is subjected to a second stress σ_2 acting perpendicular to σ_1 , as illustrated in Fig. 3b, the result is not only an elongation in direction 2, but also a reduction of the elongation in direction 1. This example

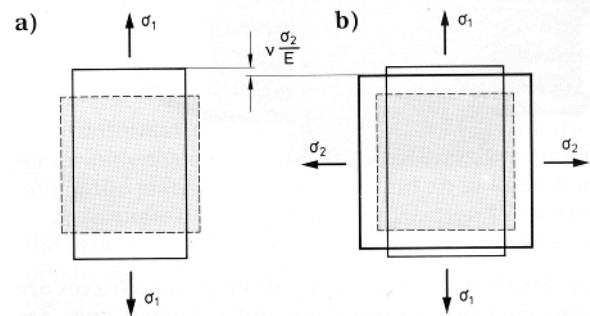


Fig. 3: Deformation of a plate due to principal stresses

- Longitudinal elongation and lateral contraction resulting from a principal stress σ_1
- Additional action of the principal stress σ_2 causes not only an elongation in direction 2, but also a reduction of the elongation in direction 1.

illustrates that biaxial stress involves the superposition of transverse and longitudinal strains. Therefore, Hooke's Law as formulated for uniaxial stress is not valid for multiaxial stress because it does not allow for lateral deformation. The component of deformation in direction 1 produced by stress σ_2 is

$$\epsilon_{1,\sigma_2} = -\nu \frac{\sigma_2}{E} \quad (2)$$

and must be subtracted from the elongation produced by σ_1 to come up with the total deformation in direction 1

$$\epsilon_1 = \frac{\sigma_1}{E} - \nu \frac{\sigma_2}{E}. \quad (3)$$

Thus for the biaxial stress state, the strain in direction 1 and direction 2 can be written as:

$$\epsilon_1 = \frac{1}{E} (\sigma_1 - \nu \sigma_2), \quad (4)$$

$$\epsilon_2 = \frac{1}{E} (\sigma_2 - \nu \sigma_1). \quad (5)$$

The result of solving equations (4) and (5) for σ_1 and σ_2 respectively is Hooke's Law for the biaxial stress state, which is the fundamental relationship for stress analysis using strain gages in the elastic deformation range:

$$\sigma_1 = \frac{E}{1 - \nu^2} (\epsilon_1 + \nu \epsilon_2), \quad (6)$$

$$\sigma_2 = \frac{E}{1 - \nu^2} (\epsilon_2 + \nu \epsilon_1). \quad (7)$$

This form of Hooke's Law for the biaxial stress state is written using stress and strain subscripts 1 and 2 to indicate the principal stress and strains for the stress state under consideration. The principal stresses are the extreme stresses, i.e. the largest and smallest

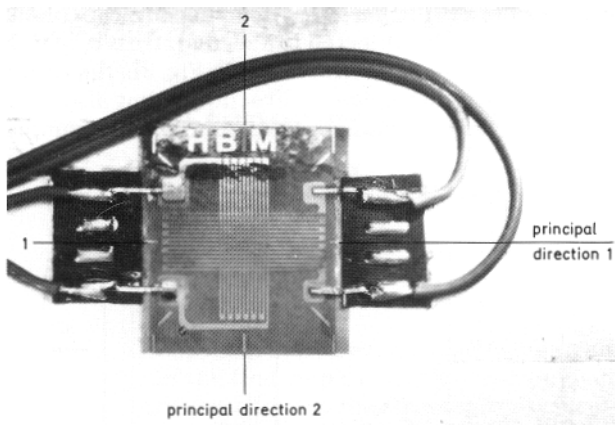


Fig. 4: Strain gage installation if the principal axes are known: measuring directions and principal axes are identical

stresses. Both these stresses are perpendicular to one another in the principal axes 1 and 2. Conventionally $\sigma_1 \geq \sigma_2$

The goal of stress analysis is to determine the principal normal stresses and principal axes. This procedure is greatly simplified if the principal axes are known before the analysis, since this leaves only the principal stresses to be determined. This can be done by orienting two strain gages as shown in Fig. 4 so that one grid measures the principal strain ϵ_1 in principal axis 1 and the second grid measures the principal strain ϵ_2 in principal axis 2. The principal normal stresses can then be calculated by entering into equations (6) and (7) the principal strains ϵ_1 , ϵ_2 , Young's Modulus E , and Poisson's Ratio ν .

Unfortunately, in most cases the principal axes of stress in the surface to be measured are unknown, so that mounting of strain gages aligned to the principal axes is not possible and the principal strains cannot be measured directly. Determination of the principal axes is therefore often a significant part of the stress analysis. The following explanation of the fundamentals of this part of the stress analysis should simplify understanding of the applied equations.

Stress and deformation circles

In the planar stress state, stresses occur not only in the principal axes 1 and 2, but also in other arbitrary directions within this plane. This can be viewed as a planar stress field in which the principal stress σ_1 is a maximum value and the principal stress σ_2 is a minimum value. As an aid for the description of the arbitrarily oriented stresses in this plane, a Cartesian coordinate system is used which is rotated around the principal axes 1 and 2 by the arbitrary angle φ .

If we were to cut a small surface element from the plate shown in Fig. 5 and observe the stresses released by doing so, we would find stresses σ_x and σ_y acting normal to the sides of the cut, as well as shear stresses τ acting parallel to the cuts. Only when the cut surfaces are oriented with principal stresses σ_1 and σ_2 are they free from shear stress. Applying equilibrium conditions on

the element shown in Fig. 5, the relationship between principal stresses and stresses in the cut surfaces can be determined based on the angle φ . This procedure is explained in many textbooks and therefore will not be described here in greater detail.

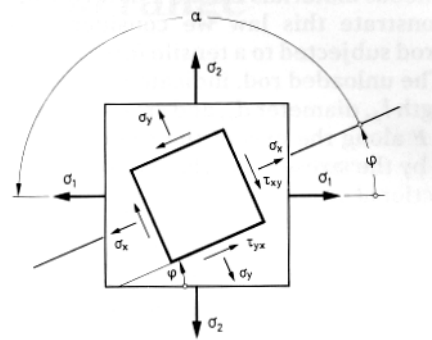


Fig. 5: Biaxial stress state, principal stresses σ_1 and σ_2 and the stresses at any arbitrary angle φ

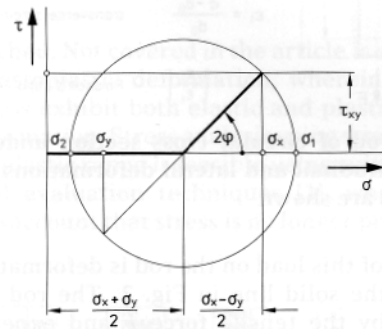


Fig. 6: Mohr's Circle: a graphical representation of the relationship between principal stresses and stresses acting in arbitrary directions

A handy graphical aid for illustrating the relationship between the principal stresses and the stresses occurring in arbitrary directions is Mohr's Circle [2]. As shown in Fig. 6, it is drawn on an orthogonal σ - τ -coordinate system. Its center lies on the abscissa and intersects the σ axis at the principal normal stresses σ_1 and σ_2 . When a straight line is drawn through the center of the Mohr Circle at an angle 2φ , the points of intersection on the circle graphically give the values for τ_{xy} , σ_x and σ_y . Recalling that φ is the angle in which the x - y coordinate system has been rotated around the principal axes 1 and 2, the same method can be reversed such that the principal normal axes and φ can be determined when the stresses τ_{xy} , σ_x and σ_y are known. From Fig. 6 it is easy to obtain the equation for the calculation of the principal normal stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2} \quad (8)$$

as well as for the angle

$$\varphi = \frac{1}{2} \arctan \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} \quad (9)$$

By setting the condition that $\sigma_1 \geq \sigma_2$ and $\sigma_x \geq \sigma_y$ the ambiguity of the tangent function in equation (9) can be eliminated, i.e. $0 \leq \varphi \leq 45^\circ$. The angle φ describes the direction of the x axis and is rotated in a mathematically positive direction starting from the principal axis 1.

It can be seen from Mohr's Circle that the maximum shear stresses occur when φ is 45° and that the surfaces normal to the principal axes contain no shear stress. In the case of pure torsion, the centre of the Mohr Circle coincides with the origin of the coordinates and the principal normal directions are $\sigma_1 = -\sigma_2 = \tau_{max}$, whereby the principal directions are oriented 45° to the axis of the torsional moment.

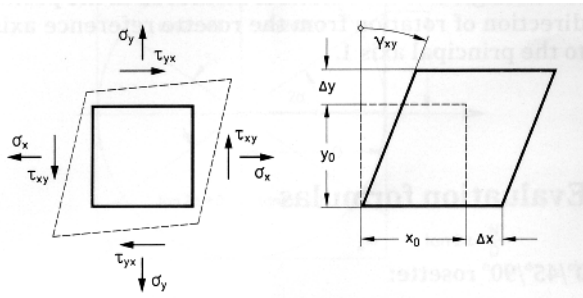


Fig. 7: Deformation of a small element created by normal and shear stresses

Although all materials under stress experience three-dimensional deformation, for stress analysis with strain gages, only the stresses existing in the material surface are of importance. Illustrated in Fig. 7 are the possible stresses and deformations which could occur in a small element cut from the surface of a material under stress. The normal stresses σ_x and σ_y produce the strains ϵ_x and ϵ_y causing length changes in the x - and y directions. The shear stresses τ_{xy} and τ_{yx} produce shear deformation, causing a change in angle γ_{xy} .

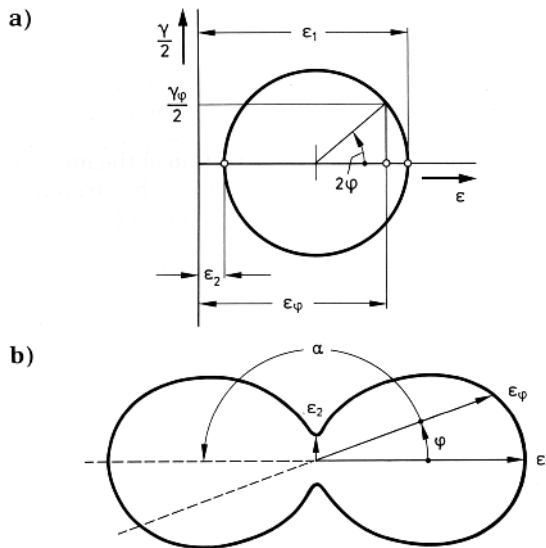


Fig. 8: Graphical representation of the deformation condition in the specimen's surface
a) Deformation circle
b) Planar stress field corresponding to the deformation circle

Analogous to the Mohr Circle method for graphically determining the stresses in arbitrary directions, strains and distortion angles are illustrated using a deformation circle [41]. As shown in Fig. 8a, this circle is represented in deformation coordinates, whereby the strain F is used as the abscissa, and the distortion angle $\gamma/2$ is used as the ordinate. The center of the deformation circle lies on the abscissa, and its points of intersection with the abscissa determine the principal strains ϵ_1 and ϵ_2 .

The deformation circle provides an easy method to determine the strains occurring in arbitrary directions along the material surface. To find an unknown strain ϵ_φ at an angle φ to the principal direction 1— assuming the principal strains and directions are known — a perpendicular is dropped to the abscissa from a point on the circle where the radius at 2φ intersects it (Fig. 8a). Fig. 8b illustrates in polar coordinates the planar strain field corresponding to the deformation circle shown in Fig. 8a. The significance here is that in both illustrations the angle φ or 2φ respectively is turned in a positive rotational sense from the same reference direction.

From the deformation circle shown in Fig. 8a the relationship between the effective strain ϵ_φ and the principal strains ϵ_1 and ϵ_2 can be derived:

$$\epsilon_\varphi = \frac{\epsilon_1 + \epsilon_2}{2} + \frac{\epsilon_1 - \epsilon_2}{2} \cos 2\varphi \quad (10)$$

Equation (10) provides the key to experimental stress analysis using strain gages, assuming the principal directions of stress are unknown. It contains as unknowns the principal strains ϵ_1 and ϵ_2 as well as the angle φ , assuming that the strain in any arbitrary direction φ can be obtained by measurement. Measuring the strain in three different directions at the surface of a specimen will give three equations (10) to determine the three unknowns. This method assumes that the angles between the measuring axes are known, which is the case when strain-gage rosettes are used.

Types of rosettes and grid notation

Figure 9 shows a strain-gage rosette with three measuring grids cemented to a specimen in preparation for stress-analysis. The measuring directions of the grids

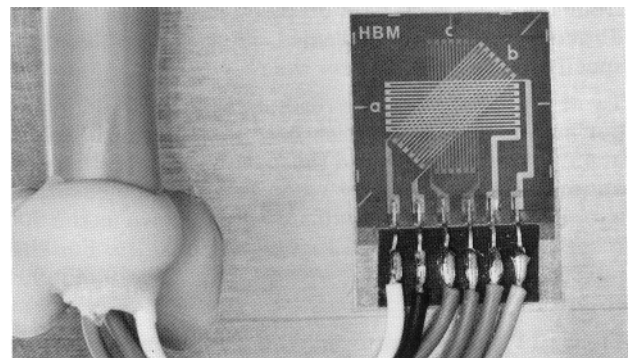


Fig. 9: A strain-gage rosette attached to a specimen in preparation for stress analysis

are oriented 45° to one another. A strain-gage rosette containing three measuring grids provides three separate readings which, with the aid of mathematical evaluation, can be used to determine the principal strains and their directions at the measuring point. The mathematical formulas required for this evaluation depend on the type of rosette used as well as the notation of its measuring grids. Various methods of measuring grid notation for rosettes with the same grid arrangement can be found. This leads to different formulation of equations for the principal strains and their directions. These equations have no universal validity, but must always be viewed in connection to the method of notation employed in the derivation of the equation.

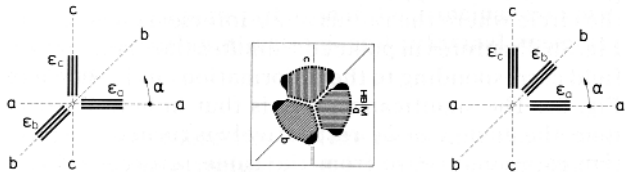


Fig. 10: Configuration and notation of the measuring axes of 0°/45°/90° strain-gage rosettes

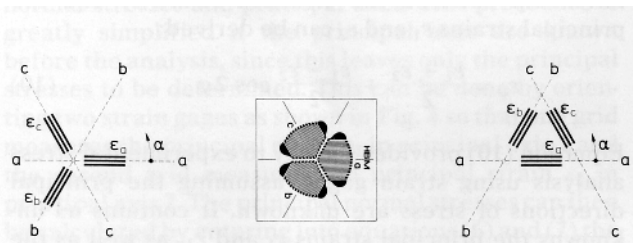


Fig. 11: Configuration and notation of the measuring axes of 0°/60°/120° strain-gage rosettes

Errors are commonly encountered in determining the directions of the principal axes because of the various ways of defining the rotational sense of the angle between measuring directions and principal axes. According to Fig. 6 and 8, the angle φ is measured starting at the principal axis 1, rotating in a positive sense to the reference axis. However, when carrying out measurements using rosettes, the inverse procedure must be followed, i.e. the measurement directions are known and the principal direction must be found. Therefore, it is practical to introduce an angle α which is rotated from the reference axis of the rosette in a positive direction to the principal axis 1. Thus, it is necessary to specify a reference axis for the rosette.

Two types of strain-gage rosettes are preferred in experimental stress analysis: those with measuring axes at 45° to one another, and those with axes at 60° to one another. Figure 10 shows rosettes with the measuring axes at 45°. This grid arrangement is known as the 90° rosette, 45° rosette, and rectangular rosette. For the purposes of this article, we will call it the 0°/45°/90° rosette.

Figure 11 shows rosettes in which the measuring axes are at 60°. This we will call the 0°/60°/120° rosette. It is otherwise known as the 0°/120°/240° rosette, delta rosette, and equiangular rosette.

In the following section, evaluation formulas are introduced for the 0°/45°/90° and 0°/60°/120° rosettes. The measuring axes for the individual grids are denoted consecutively by the letters a , b , and c in the positive direction of rotation. The reference axis of the rosette is designated "a". Using the 0°/45°/90° rosette, the reference axis will be the first measurement axis encountered when moving in a positive direction of rotation. The principal axes are given the notation 1 and 2, in conformance with normal practice in materials strength analysis. The direction notation is employed in the form of subscripts for the different strain components, whereby the measured strains are ϵ_a , ϵ_b and ϵ_c and the principal strains are ϵ_1 and ϵ_2 , where $\epsilon_1 \geq \epsilon_2$. The orientation angle α of the rosette is measured in the positive direction of rotation from the rosette reference axis a to the principal axis 1.

Evaluation formulas

0°/45°/90° rosette:

By applying equation (10) for the readings obtained from a 0°/45°/90° rosette, it is possible to obtain the defining equations for the principal strains ϵ_1 and ϵ_2 as well as the orientation angle α [4]. These equations are:

$$\epsilon_{1,2} = \frac{\epsilon_a + \epsilon_c}{2} \pm \frac{1}{2} \sqrt{2 \sqrt{(\epsilon_a - \epsilon_b)^2 + (\epsilon_c - \epsilon_b)^2}} \quad (11)$$

and

$$\alpha = \frac{1}{2} \arctan \frac{2\epsilon_b - \epsilon_a - \epsilon_c}{\epsilon_a - \epsilon_c} \quad (12)$$

or

$$\tan 2\alpha = \frac{2\epsilon_b - \epsilon_a - \epsilon_c}{\epsilon_a - \epsilon_c} \quad (13)$$

Note: equations (11) to (13) are only valid for the rosette notation indicated in Fig. 10.

0°/60°/120° rosette:

Reference [4] also contains a description of the method for deriving the defining equations for the principal strains ϵ_1 and ϵ_2 and the orientation angle for the 0°/60°/120° rosette. They are:

$$\epsilon_{1,2} = \frac{\epsilon_a + \epsilon_b + \epsilon_c}{3} \pm \sqrt{\left(\frac{2\epsilon_a - \epsilon_b - \epsilon_c}{3}\right)^2 + \frac{1}{3}(\epsilon_b - \epsilon_c)^2} \quad (14)$$

and

$$\alpha = \frac{1}{2} \arctan \frac{\sqrt{3}(\epsilon_b - \epsilon_c)}{2\epsilon_a - \epsilon_b - \epsilon_c} \quad (15)$$

or

$$\tan 2\alpha = \frac{\sqrt{3}(\epsilon_b - \epsilon_c)}{2\epsilon_a - \epsilon_b - \epsilon_c} \quad (16)$$

Note: equations (14) to (16) are only valid for the rosette notation indicated in Fig. 11.

Ambiguity of the tangent function

Equations (13) and (16) supply the orientation angle of the principal axis 1 for the 0°/45°/90° and 0°/60°/120° rosettes. With both types, the orientation angle α is obtained from a tangent function of 2α which always gives an ambiguous result because $0^\circ \leq 2\alpha \leq 360^\circ$. The following is a brief mathematical explanation meant to prevent errors caused by the ambiguity of the tangent function.

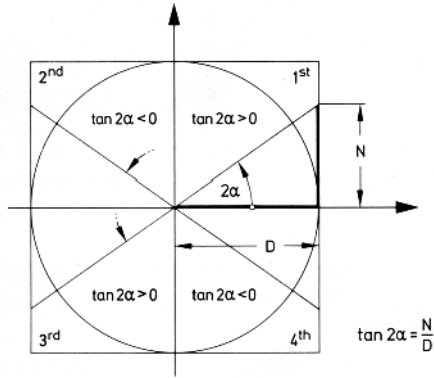


Fig. 12: Quadrant locator for unambiguous location of the tangent angle

The tangent of an angle is a quotient. As shown in Fig. 12, the numerator N is the part of the tangent (opposite side) subtended by the angle 2α and the denominator D is the radius (adjacent side). In the first quadrant, numerator and denominator are positive, so the quotient is also positive. In the third quadrant, the quotient is again positive, but the numerator and denominator are negative. From this the ambiguity of the tangent function throughout the range $0^\circ \leq \alpha \leq 360^\circ$ is easily seen.

For a clear determination of 2α , it is necessary to examine the sign of the numerator N and the denominator D . This is possible using Table 1, which determines unambiguously the angle α . This angle can then be measured from the rosette reference axis "a" in a positive sense of rotation. This fixes the principal axis 1. Principal axis 2 is perpendicular to principal axis 1.

Example of stress analysis using a 0°/60°/120° strain gage rosette

In order to demonstrate the application of the derived formulae, an example stress analysis will be performed using actual measurement values. The demonstration involves the mounting of a 0°/60°/120° rosette on a specimen to determine the planar stress state at the measuring point.

The first step following the application of the rosette to the unstressed specimen is to ascertain the reference axis of the rosette and to label the individual measuring axes according the guidelines previously laid out. Figure 13 shows the rosette and relevant notation. A point of practical importance is that the wiring from the rosette should be clearly marked with the same

N	$\cong 0$	> 0	$\cong 0$	< 0
D	> 0	$\cong 0$	< 0	$\cong 0$
2α	$\arctan \frac{N}{D}$	$\pi - \arctan \frac{N}{D}$	$\pi + \arctan \frac{N}{D}$	$2\pi - \arctan \frac{N}{D}$
α	$0^\circ \cong \alpha < 45^\circ$	$45^\circ \cong \alpha < 90^\circ$	$90^\circ \cong \alpha < 135^\circ$	$135^\circ \cong \alpha < 180^\circ$

Table 1: Relationship between the angle α and the values for the numerator N and the denominator D

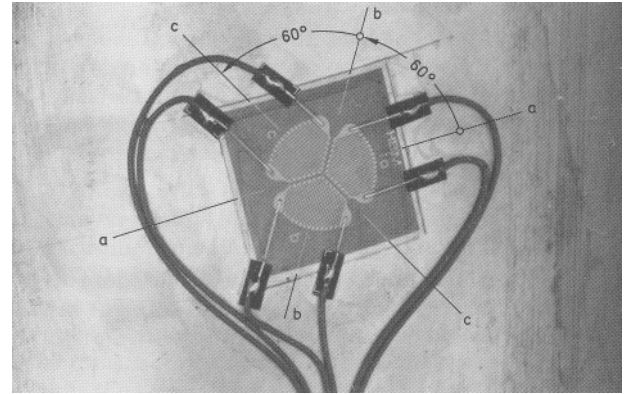


Fig. 13: Notation of the measuring axes of a strain gage rosette

notation as the individual grids in order to avoid confusion when evaluating the signals from several rosettes.

Assuming a satisfactory and error-free measurement procedure, the following three readings were obtained from the rosette on the loaded specimen:

$$\begin{aligned} \varepsilon_a &= 309 \cdot 10^{-6}; \\ \varepsilon_b &= 86 \cdot 10^{-6}; \\ \varepsilon_c &= 205 \cdot 10^{-6}. \end{aligned} \quad (17)$$

Calculation of the angle 2α for a 0°/60°/120° rosette is performed by substituting the measurement values into equation (16):

$$\tan 2\alpha = \frac{\sqrt{3}(86 - 205)}{618 - 86 - 205} = \frac{N}{D}; \quad (18)$$

$$\frac{N}{D} = \frac{-206}{327}. \quad (19)$$

Since $N < 0$ and $D > 0$, equations (18) and (19) show that the angle 2α lies in the fourth quadrant. In the fourth quadrant

$$2\alpha = 2\pi - \arctan |N/D| \quad (20)$$

and with

$$\tan |N/D| = 0,63 \quad (21)$$

and

$$\arctan |N/D| = 32^\circ \quad (22)$$

therefore,

$$2\alpha = 360^\circ - 32^\circ \quad (23)$$

$$\alpha = 164^\circ. \quad (24)$$

Figure 14 shows a graphical aid for the determination of 2α by means of the tangent function. It is obvious that without an individual examination of numerator and denominator, the angle 2α could be mistakenly placed in the second quadrant (148°), instead of correctly in the fourth quadrant (328°).

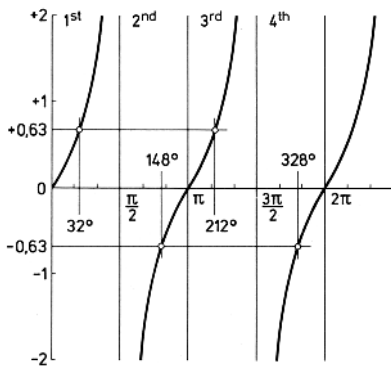


Fig. 14: Graphic representation of the tangent function. The ambiguity of determining the angle in the example is shown

Now that the orientation angle for the principal axis 1 has been determined, it can be seen that principal axis 1 is at 164° to the reference axis, measured in the positive direction of rotation. Principal axis 2 is, of course, perpendicular to principal axis 1. **Figure 15** shows the calculated orientation angle and the principal axes.

Substituting the readings from equation (17) into the defining equation (14) for the principal strains of the $0^\circ/60^\circ/120^\circ$ rosette gives:

$$\varepsilon_{1,2} = [200 \pm 129] \cdot 10^{-6}$$

$$\varepsilon_1 = 329 \cdot 10^{-6}$$

$$\varepsilon_2 = 71 \cdot 10^{-6}$$

With this, the principal strains have been established.

Through equations (6) and (7), Hooke's Law provides the necessary relationship for calculating the principal normal stresses (under the assumption of elastic deformation, which can be checked by measuring again when the specimen has been unloaded). The modulus of elasticity and Poisson's Ratio for the material under consideration are: $E = 206,000 \text{ N/mm}^2$ and $\nu = 0.3$. Substituting the principal strains ε_1 and ε_2 previously calculated, and the material parameters E and ν in equations (6) and (7) gives:

$$\sigma_1 = \frac{206000}{0,91} (329 + 0,3 \cdot 71) \cdot 10^{-6} \text{ N/mm}^2$$

$$\sigma_1 = 79 \text{ N/mm}^2$$

$$\sigma_2 = \frac{206000}{0,91} (71 + 0,3 \cdot 329) \cdot 10^{-6} \text{ N/mm}^2$$

$$\sigma_2 = 39 \text{ N/mm}^2$$

Thus, both the principal axes and principal stresses are now known. This provides a clear picture of the plane

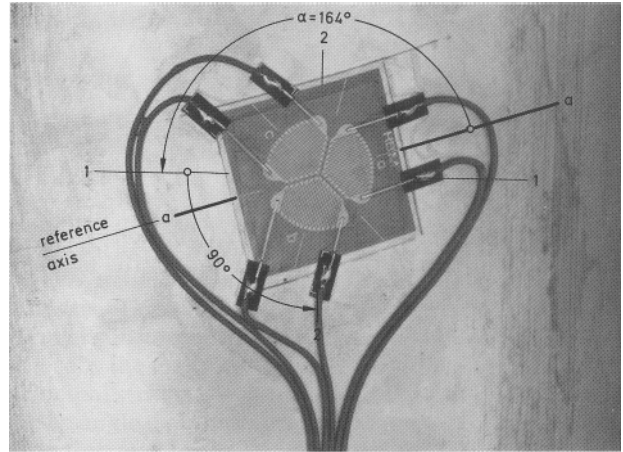


Fig. 15: Fixing the calculated principal axis 1 by measuring the angle in a positive rotation from the rosette reference axis "a"

stress at the point of measurement and completes the demonstration of experimental stress analysis using strain gages.

In closing, a clarification should be made to show the effect of transverse strain (Poisson's Ratio) on the principal strains and principal stresses at the measuring point. Note that the principal normal stress σ_1 is twice as large as σ_2 . However, the principal strain ε_1 is more than four and a half times as large as ε_2 . The transformation of strain into stress without consideration of the effect on the transverse strain can therefore lead to large errors. There is a distinct possibility of making a wrong estimation of the actual stress condition if a hurried conclusion is made.

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