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Mechanical and thermal strain

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The change in the length of a solid relative to its original length is, in a finite presentation, designated as technical strain ©in accordance with **equation 1** and in a differential presentation, designated as natural strain ♦, in accordance with **equation 2** (see [1]).

$$\varepsilon = \frac{l_1 - l_0}{l_0} = \frac{\Delta l}{l_0} \qquad (1)$$

$$\varphi = \int_{l_0}^{l_1} \frac{dl}{l} \qquad (2)$$

If you observe the physical cause of this purely geometrically defined quantity, it is possible to establish that exposure both to the effects of external force and to changes in temperature result in strain. This is designated mechanical or thermal strain, depending on its cause. These two components often occur together and displacement and strain sensors that work purely geometrically cannot evaluate or measure them separately. On the other hand, in experimental stress analysis, when defining the reaction of components to external mechanical forces or in weighing technology, when the mass of a body is derived from its force due to weight at a known gravitational constant, it is important to define only the mechanical strain.

Thermal strain

When the temperature rises in an independent solid, it expands evenly in all directions **(Fig. 1)**. On condition that there is equal access in all directions, the extent of this thermal expansion is described by:

$$\varepsilon_x^{th} = \varepsilon_y^{th} = \varepsilon_z^{th} = \alpha^{th} \cdot \Delta T \qquad (3)$$

and in many cases, this condition is easily met.

The linear expansion coefficient $/^{th}$, which is assumed to be constant here, is dependent on the material involved and is within the range $10^{-6} \le \alpha^{th} \le 10^{-4}$.

If you observe the overall expansion of crystalline materials (e.g. metals) from absolute zero T_0 to melting point T_S, this produces a value which is virtually the same for many materials, of:

$$\varepsilon_{gesamt}^{th} \left| T_0 \Rightarrow T_S \right| = 0.02 \qquad (4)$$

So it is possible, in the form of **equation 5**, to specify a relationship between the expansion coefficients and the melting temperature of a metal in °C:

$$\alpha^{th} = \frac{0.02}{T_S + 273} \qquad (5)$$

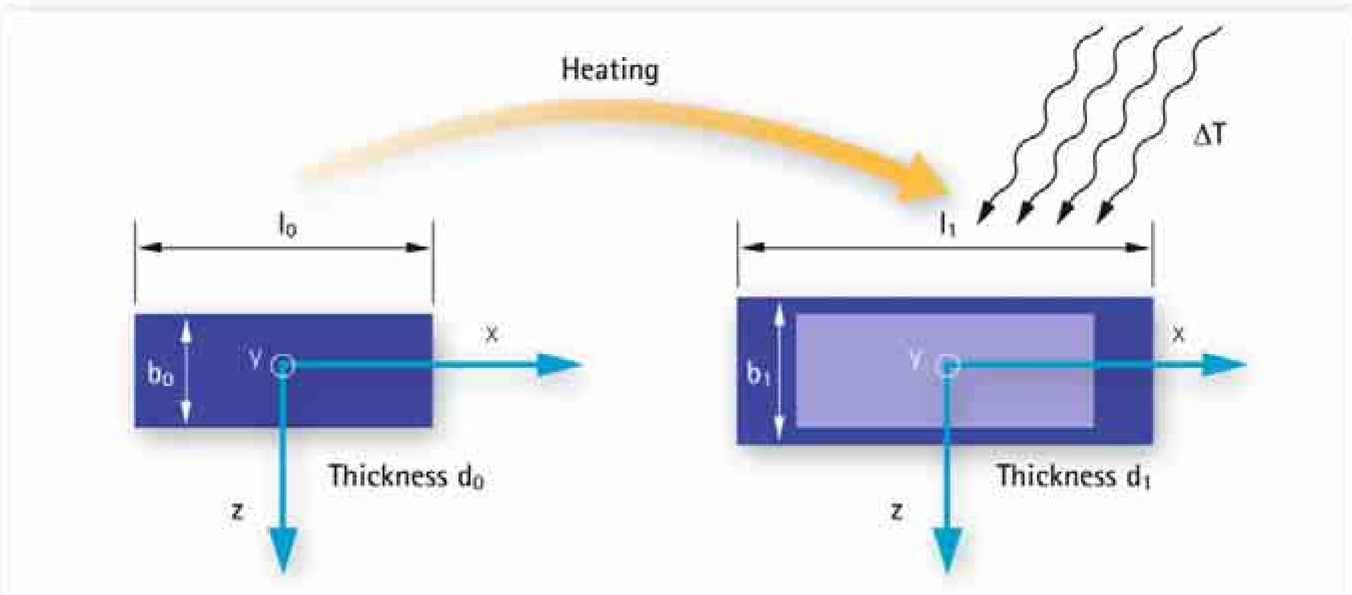


Fig. 1: Thermal strains as a result of heating

Figure 2 shows this correlation, initially described by Eduard Grüneisen, by means of the measured values of four metals to which **equation 5** applies.

Mechanical strain

If solids are put under stress by external forces, their shape changes. That is to say, there are changes in the length and angle of the original shape subject to the load and to the properties of the material. This is shown in **Figure 3** for the simple load scheme of elastic deformation in a tension bar.

The mechanical strains for this situation are defined by:

$$\varepsilon_x^{mech} = \frac{\sigma_x}{E} = \frac{F}{E \cdot A} \tag{6}$$

... and ...

$$\varepsilon_y^{mech} = \varepsilon_z^{mech} = -\nu \cdot \varepsilon_x^{mech} \tag{7}$$

... where E is the modulus of elasticity and ↓ is Poisson's ratio. This allows the external force F to be calculated from the mechanical strain.

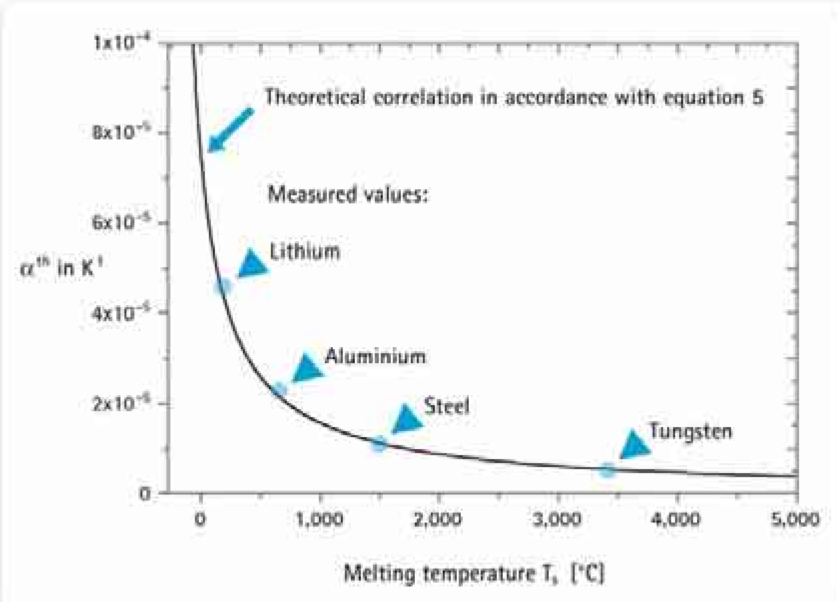


Fig. 2: Correlation between the thermal expansion coefficient/ th and the melting temperature T_S

Separate evaluation of the mechanical strain

As the thermal and the mechanical strain are cumulative, it is difficult to evaluate the mechanical strain separately. In SG measurement technology, two different methods of compensation are applied.

The first method of compensation utilizes a property of the WHEATSTONE bridge circuit that is used to measure the change in resistance of a strain gage **(Fig. 4)**. The relative changes in resistance of the individual resistances arranged there appear in positive or negative pairs in bridge **equation 8**.

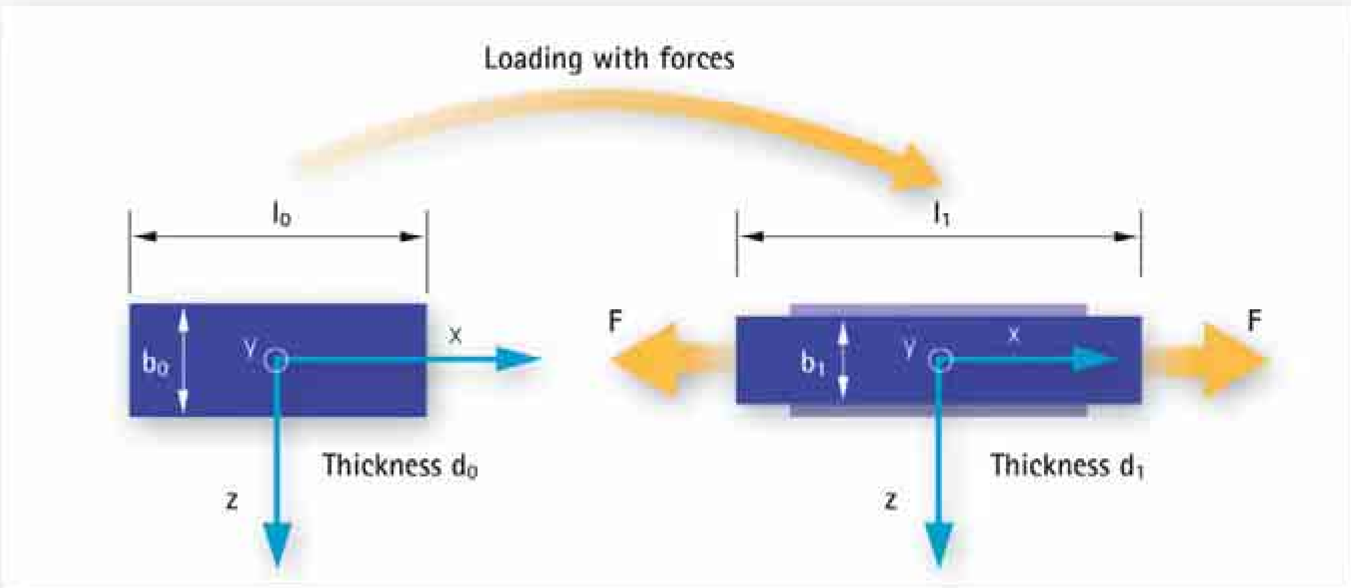
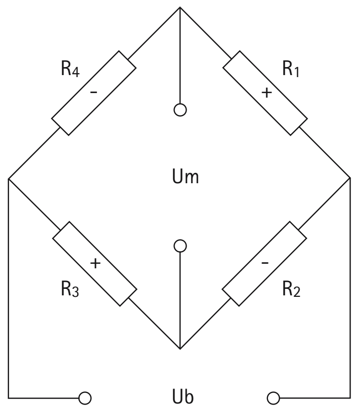


Fig. 3: Mechanical strains as a result of external forces



$$\frac{Um}{Ub} = \frac{1}{4}\left(\frac{\Delta R_1}{R_0} - \frac{\Delta R_2}{R_0} + \frac{\Delta R_3}{R_0} - \frac{\Delta R_4}{R_0}\right) \quad (8)$$

Fig. 4: WHEATSTONE bridge circuit

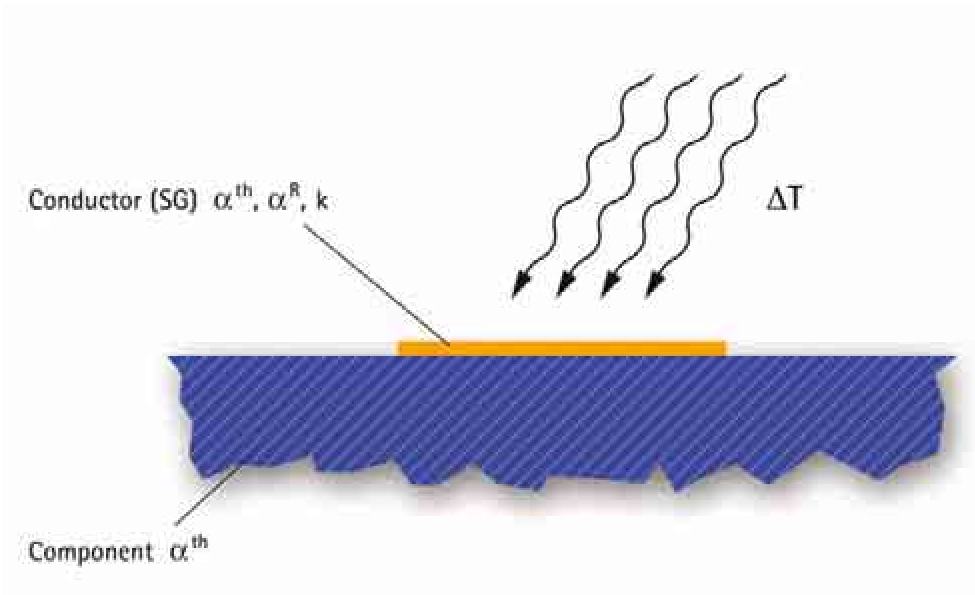


Fig. 5: Idealized representation of a conductor (SG) installed on a component, during a change in temperature

Thermal strain in all the SGs, presupposing the same temperature, always causes changes in resistance with the same mathematical sign, so compensation options are numerous. One of these options is explained here, by way of example. For example, if resistance R_1 represents the actual active SG installed on a component, it is exposed simultaneously to mechanical and thermal strain. In addition, if resistance R_2 is also implemented as an SG, but is the "dummy" on a non-mechanically loaded blank panel that is at the same temperature as the component, and if resistances R_3 and R_4 are fixed resistances, then:

$$\frac{Um}{Ub} = \frac{1}{4}\left(\frac{\Delta R_1^{mech}}{R_0} + \frac{\Delta R_1^{th}}{R_0} - \frac{\Delta R_2^{th}}{R_0}\right) \quad (9)$$

Presupposing that the thermal expansion coefficients of the component and the blank plate are also the same and that the SG has identical properties, then:

$$\frac{\Delta R_1^{th}}{R_0} = \frac{\Delta R_2^{th}}{R_0} \quad (10)$$

... and with ...

$$\frac{Um}{Ub} = \frac{1}{4}\left(\frac{\Delta R_1^{mech}}{R_0}\right) \quad (11)$$

... the thermal strain is fully compensated.

A second compensation, not connected in any way with the WHEATSTONE bridge circuit, arises from adapting the mechanical and electrical properties of the conductor material to the thermal expansion coefficients of the component material.

Because the thermal expansion coefficients in the component and in the conductor are different **(Fig. 5)**, when the temperature of the overall system changes, there is mechanical strain in the conductor:

$$\varepsilon_L^{mech} = \varepsilon_B^{th} - \varepsilon_L^{th} = \left(\alpha_B^{th} - \alpha_L^{th}\right) \Delta T \quad (12)$$

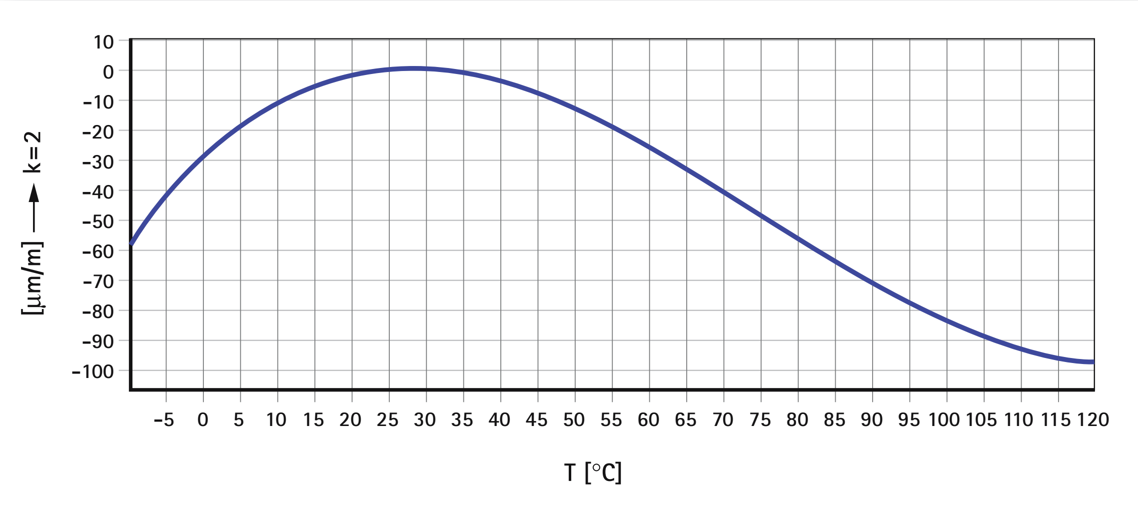
Certainly the conductor itself has a temperature response even in a mechanically unloaded state, which can be expressed by ...

$$\frac{\Delta R}{R_0} = \alpha^R \Delta T \quad (13)$$

... where ⁄ ^{R} is the temperature coefficient of the resistance. It therefore follows with ...

$$\frac{\Delta R}{R_0} = \left[k\left(\alpha_B^{th} - \alpha_L^{th}\right) + \alpha^R\right] \Delta T \quad (14)$$

... that the resistance of a conductor prepared on a component changes when there is a change in temperature.



$$\varepsilon_S (T) = -28.3 + 2.33 \cdot T - 5.19 \cdot 10^{-2} \cdot T^2 + 2.31 \cdot 10^{-4} \cdot T^3 \pm 0.3\ (\mu m/m)\ °C^{-1}$$

Fig. 6: Graphical and analytical representation of the temperature response of the LY41-6/120 strain gage, adapted to steel with $\alpha_B^{th} = 10.8 \cdot 10^{-6} K^{-1}$

On the condition that this thermally induced fraction of the total change in resistance is to disappear, it is possible to specify a formal correlation between the thermal expansion coefficient of conductor and component and a requisite temperature coefficient of the resistance:

$$\frac{\Delta R}{R_0} = 0 \quad \Rightarrow \quad \alpha^R = k\left(\alpha_L^{th} - \alpha_B^{th}\right) \qquad (15)$$

SG resistance materials such as constantan or Karma alloy have the property that ⁄ ^{R} can be set by a defined heat treatment of the cold-rolled foil. This makes it possible to comply with **equation 15** for different component materials.

All three of the temperature coefficients $\left(\alpha^R,\ \alpha_L^{th},\ \alpha_B^{th}\right)$ are only constants within a narrow range of temperatures, so that the temperature adaptation obtained for a component material only ever applies to this temperature range.

Strain gage manufacturers therefore specify this temperature adaptation as a curve and frequently as a polynomial as well. This information is displayed in **Figure 6** for HBM's LY41-6/120 strain gage, adapted to steel with $\alpha_B^{th} = 10.8 \cdot 10^{-6} K^{-1}$.

This clearly shows that with an initial temperature of 20 °C and an increase in temperature of 20 K to 40 °C, the display is restricted to $\varepsilon_S \approx 3 \mu m/m$.
On the other hand, the thermal strain of the component is $\varepsilon_B^{th} = 216 \mu m/m$.

In an extended temperature range, it is possible to reduce the measurement error resulting from the remaining residual temperature response by using the specified polynomial.

The compensation methods described here reflect the advanced development status of strain gage technology and are a considerable help in minimizing measurement uncertainty in experimental stress analysis and in transducers. The two methods are frequently used in combination and can also be supplemented by additional compensation measures (see [2]), especially for demanding requirements, as is often the case with transducers.

References

[1] Martin Stockmann: Technische und natürliche Dehnung, MTB 2/2003, pages 18-19

[2] HBM house journal: Der Weg zum Messgrößenaufnehmer – ein Leitfaden zur Anwendung der HBM K-DMS und der Zubehörkomponenten