From measured strain to mechanical stress …

Analysis of the biaxial stress state with unknown principal directions

Published on: http://www.hbm.com/en/menu/anwendungen/experimentelle-spannungsanalyse/technische-fachartikel/esa-technical-articles-detail-view-de/datum/2008/06/24/from-measured-strain-to-mechanical-stress-analysis-of-biaxial-stress-state-with-unknown-principal-d/

The principle of experimental stress analysis using strain gages consists in using strain gages to measure strains on the component surface. From these measured strains and the known material properties such as modulus of elasticity and Poisson's ratio, the absolute value and the direction of these mechanical stresses are determined. These calculations are based on **Hooke's Law** which applies to the elastic deformation range of linear-elastic materials.

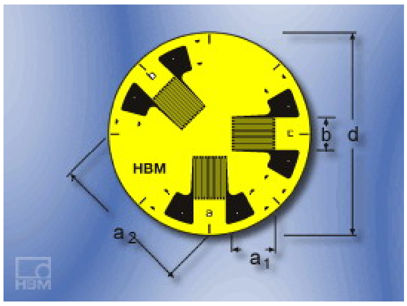
In experimental stress analysis, so-called 3-grid rosettes are used for strain measurement. These are available in 0°/45°/90° and 0°/60°/120° versions which have a historical background. It is up to the user to choose which version to use.

The 3 measuring grids of the rosettes are designated with the letters a, b and c. Therefore, a 3-grid rosette measures the three strains εa, εb and εc.

The principal normal stresses σ1 and σ2 for the 0°/45°/90° rosette are calculated as follows:

:

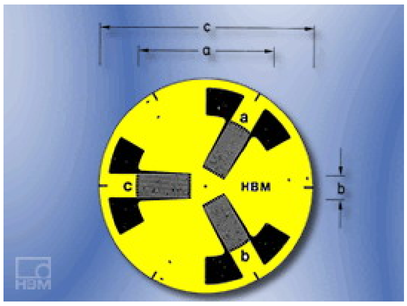
$$\sigma_{1/2} = \frac{E}{1-\nu}\cdot\frac{\varepsilon_a+\varepsilon_c}{2} \pm \frac{E}{\sqrt{2}(1+\nu)}\cdot\sqrt{(\varepsilon_a-\varepsilon_b)^2+(\varepsilon_c-\varepsilon_b)^2}$$



0°/45°/90° rosette, e.g. RY3x

and for the 0°/60°/120° rosette:

$$\sigma_{1/2} = \frac{E}{1-\nu}\cdot\frac{\varepsilon_a+\varepsilon_b+\varepsilon_c}{3} \pm \frac{E}{1+\nu}\cdot\sqrt{\left(\frac{2\varepsilon_a-\varepsilon_b-\varepsilon_c}{3}\right)^2+\frac{1}{3}(\varepsilon_b-\varepsilon_c)^2}\ .$$



0°/60°/120° rosette,e.g. RY7x

Below, the principal directions are determined. First the tangent of an auxiliary angle ψ is calculated.

For the 0°/45°/90° rosette according to the formula:

$$\tan\psi = \frac{2\varepsilon_b-\varepsilon_a-\varepsilon_c}{\varepsilon_a-\varepsilon_c} \qquad \left|\ \frac{N}{D}\right.$$

and for the 0°/60°/120° rosette according to the formula:

$$\tan\psi = \frac{\sqrt{3}(\varepsilon_b - \varepsilon_c)}{2\varepsilon_a - \varepsilon_b - \varepsilon_c} \qquad \Big| \qquad \frac{N}{D}$$

Note: The tangent of an angle in the right-angled triangle is the ratio of the opposite side (numerator N) to the adjacent side (denominator D):

$$\tan\psi = \frac{oposite\ side}{adjacent\ side} = \frac{N}{D}$$

This ambiguity of the tangent makes it necessary to determine the signs of the numerator (N) and the denominator (D) before carrying out the final calculation of the two above mentioned quotients. Determining the signs is important because they alone indicate the quadrant of the circular arc in which the angle ψ is located.

From the numerical value of the tangent the size of the angle ψ should then be found:

$|\psi| = \text{arctan}\ [°]$.

Then the angle φ should be determined using the below scheme:

$$\left.\begin{matrix} N \geq 0\ (+) \\ D > 0\ (+) \end{matrix}\right\} \quad \varphi = \frac{1}{2}\left(0^\circ + |\psi|\right)$$

$$\left.\begin{matrix} N > 0\ (+) \\ D \leq 0\ (-) \end{matrix}\right\} \quad \varphi = \frac{1}{2}\left(180^\circ - |\psi|\right)$$

$$\left.\begin{matrix} N \leq 0\ (-) \\ D < 0\ (-) \end{matrix}\right\} \quad \varphi = \frac{1}{2}\left(180^\circ + |\psi|\right)$$

$$\left.\begin{matrix} N < 0\ (-) \\ D \geq 0\ (+) \end{matrix}\right\} \quad \varphi = \frac{1}{2}\left(360^\circ - |\psi|\right)$$

The angle φ, found in this manner, should be applied from the axis of the reference measuring grid a in the mathematically positive direction (counterclockwise). The axis of measuring grid a forms one side of the angle φ, the second side gives prinicipal direction 1. This is the direction of the principal normal stress σ1 which is identical to the direction of the principal strain ε1. The point of the angle is located at the intersection of the axes of the measuring grids. The principal direction 2 (direction of the principal normal stress σ2) has the angle φ +90°.