Measurement of Symmetrical Components

in Three-Phase Systems

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HBM Test and Measurement

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Summary

The growing proportion of renewable energy requires these energy producers to also achieve high reliability and defined behavior in critical operating situations. The symmetrical components method can be used to assess and evaluate the operating points of renewable energy producers. This report explains the basic theoretical principles behind this method, and its practical implementation in a modern measurement system.

1. Introduction

The energy transition – i.e. the generation of electricity from renewable energy – is setting major challenges for energy suppliers, grid operators and system manufacturers. The large and still growing contribution of wind power to electricity generation, in particular, means that the technical requirements have to be very exacting, if a reliable energy supply is to be guaranteed.

In the development of wind turbines, manufacturers have gained a lot of experience in testing the individual components. The trialling and testing of wind turbines as a system is becoming ever more complex; firstly, because system output is continually increasing, and secondly, because the technical certification guidelines are becoming more and more exhaustive.

In order to certify the electrical characteristics of wind turbines in field tests or system test benches in accordance with the Technical Guideline TR3 [4] of the Federation of German Windpower and other Renewable Energies (FGW e.V.), a precise measurement system with highly efficient mathematical analysis is needed. In this report, we present and explain mathematical methods aimed at describing and evaluating the behavior of wind turbines at the grid feed-in point.

2. Symmetrical Three-Phase System

In a three-phase network, with its generators, cables, transformers, current converters and consumer devices, a symmetrical operating state is usually desirable. In other words, the currents and voltages in the three phases of the three-phase system have identical RMS values, and the voltages and currents have a phase shift of 120° in relation to each other. Figure 2.1 shows the equivalent circuit diagram, vector diagram and associated line chart of a symmetrical three-phase system. The phasors of the individual phase voltages and currents can be converted into one another using the three-phase operator $\underline{a} = e^{j120°}$.

$$\underline{U}_2 = \underline{U}_1 \cdot e^{-j120°} = \underline{U}_1 \cdot \underline{a}^2 \qquad \underline{U}_3 = \underline{U}_1 \cdot e^{-j240°} = \underline{U}_1 \cdot \underline{a} \tag{2.01}$$
$$\underline{I}_2 = \underline{I}_1 \cdot e^{-j120°} = \underline{I}_1 \cdot \underline{a}^2 \qquad \underline{I}_3 = \underline{I}_1 \cdot e^{-j240°} = \underline{I}_1 \cdot \underline{a}$$

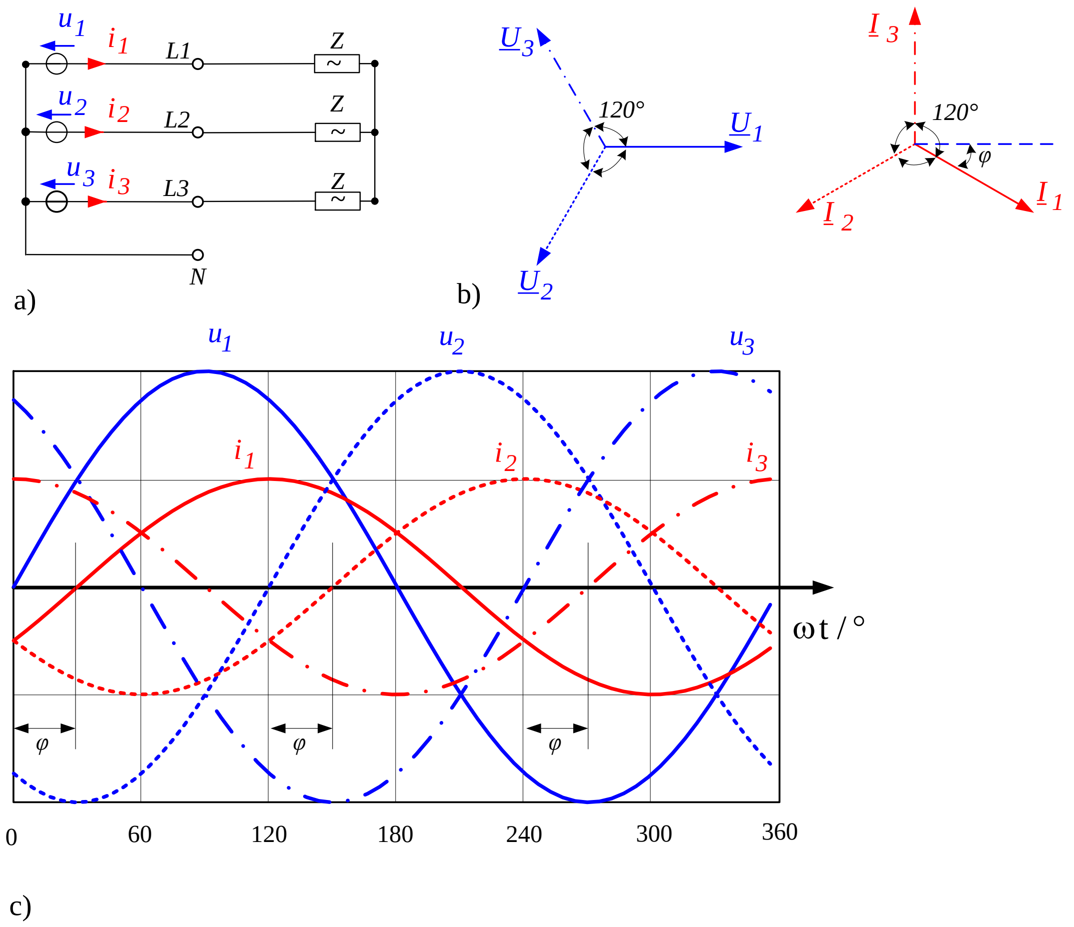

Fig. 2.1: Symmetrical three-phase system: a) Equivalent circuit, b) Phasor diagram, and c) Line chart of voltages and currents

A symmetrical three-phase system can be analyzed by means of a single-phase equivalent circuit. For example, if the impedance is known the current in phase 1 can be calculated from

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} \tag{2.02}$$

The currents and voltages in the other two phases can be calculated by multiplying them by the three-phase operators, as shown in equation 2.01.

3. Symmetrical Components

If the symmetry in a three-phase network is disrupted, e.g. due to a short circuit, ground fault or power outage, the symmetrical components method is often used to calculate and describe it. The basic idea behind symmetrical components originated from C.L. Fortescue in 1918 [1].

It is based on the mathematical principle that every asymmetrical three-phase voltage system ($\underline{U}_1$, $\underline{U}_2$ and $\underline{U}_3$) can be described by three symmetrical three-phase systems. These three symmetrical three-phase systems are known as symmetrical components, also see [2].

$$\begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \underline{a}^2 & \underline{a} & 1 \\ \underline{a} & \underline{a}^2 & 1 \end{bmatrix} \begin{bmatrix} \underline{U}_{pos} \\ \underline{U}_{neg} \\ \underline{U}_0 \end{bmatrix}$$

The symmetrical components are referred to as the positive $\underline{U}_{pos}$, negative $\underline{U}_{neg}$ and zero sequence system $\underline{U}_0$. The three-phase operator is $\underline{a} = e^{j120°}$, as mentioned above. These relationships naturally apply not just to voltages, but also to the currents in a three-phase system.

In order to break the three-phase voltages ($\underline{U}_1$, $\underline{U}_2$ and $\underline{U}_3$) down into their symmetrical components, equation 3.01 must be transformed as follows:

$$\begin{bmatrix} \underline{U}_{pos} \\ \underline{U}_{neg} \\ \underline{U}_0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_3 \end{bmatrix} \qquad (3.02)$$

3.1 Three-Phase System with Cyclic-Symmetrical Impedances

Circuit 3.1a) clearly presents the advantages of using the analysis of symmetrical components. The three voltage sources supply the circuit. These voltage sources may represent both the simple model of a three-phase generator and a converter feed. In addition, $\underline{Z}_L$ is the self-impedance of the three-phase system. The three phases are mutually coupled by coupling impedance $\underline{Z}_K$. The neutral impedance $\underline{Z}_E$ is the impedance in the neutral conductor.

First of all, the source voltages are represented by symmetrical components. For this purpose, simultaneous equation 3.01 is illustrated by an equivalent circuit diagram. By applying these equations in formal methods, we obtain the voltage $\underline{U}_1$ by connecting the voltage sources $\underline{U}_0$, $\underline{U}_{pos}$ and $\underline{U}_{neg}$ in series. Accordingly, voltage $\underline{U}_2$ is composed of $\underline{U}_0$, $\underline{a}^2\underline{U}_{pos}$ and $\underline{a}\,\underline{U}_{neg}$. Likewise, voltage $\underline{U}_3$ is the sum of $\underline{U}_0$, $\underline{a}\underline{U}_{pos}$ and $\underline{a}^2\,\underline{U}_{neg}$. To clarify this further, this circuit can be interpreted as the series connection of an AC generator ($\underline{U}_0$) and a clockwise three-phase generator that forms the positive sequence system, and an anti-clockwise three-phase generator that represents the negative sequence system.

Now we can apply the superposition theorem, as with every linear system.

Here, the voltages of two systems are always set to negative, enabling a simple calculation of the current that flows due to the third system. In this way, three equivalent circuits as shown in Figure 3.1c) are obtained.

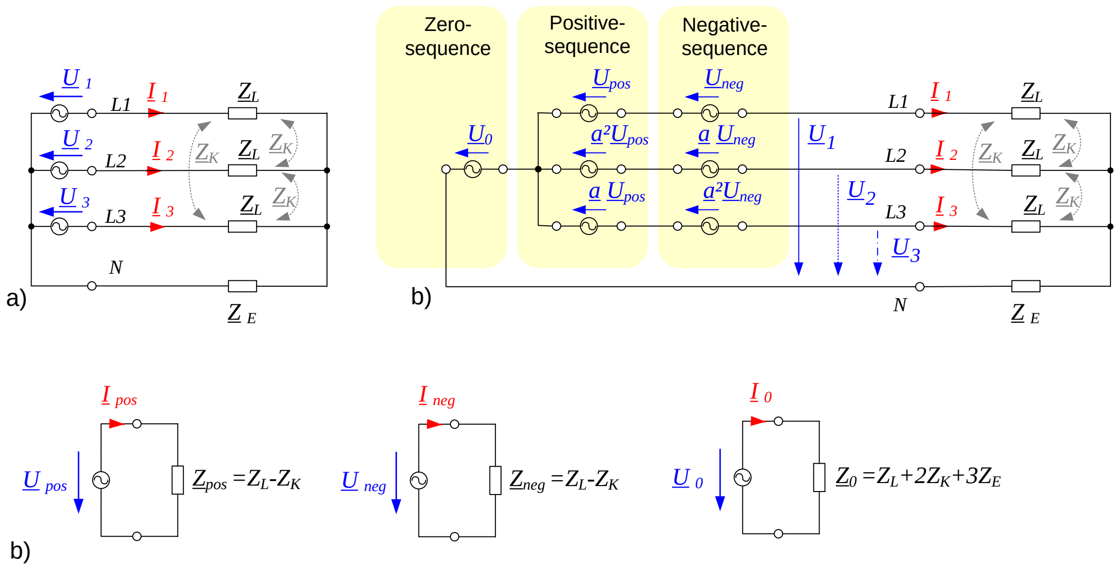

Fig. 3.1: Description of a three-voltage source with cyclic-symmetrical load impedances through symmetrical components

Positive sequence system: The circuit of the positive sequence system concurs with the single-phase equivalent circuit diagram of the symmetrical three-phase system. If the feed is symmetrical, the positive sequence component of the input voltage corresponds to voltage $\underline{U}_1$. The impedance of the positive sequence system is calculated from the difference between self-impedance and coupling impedance. The impedance of the positive sequence system is not dependent on the neutral impedance $\underline{Z}_E$, as this system does not produce any current via the neutral conductor in the case of cyclic-symmetrical impedance. Therefore, where the impedance in the positive sequence system is concerned, it is irrelevant whether the star points are grounded or not.

Elements connected in a triangle must first be converted into star connections. In the symmetrical three-phase network, all star points have the same potential, regardless of whether or not they are interconnected.

Negative sequence system: The equivalent circuit diagram basically has the same layout as the positive sequence system. With a symmetrical feed, the negative sequence component equals zero $(\underline{U}_{neg} = 0)$. In a cyclic-symmetrical network, the negative sequence impedance is the same as the positive sequence impedance.

Zero sequence system: The zero sequence system is suppled by the zero sequence component of the three-voltage source. With symmetrical three-phase voltages, the negative sequence component equals zero $(\underline{U}_0 = 0)$. The way in which the star points are connected is of fundamental importance in the zero sequence system. Currents in the zero sequence system must flow through the star connections. If the star points of the source and load are not interconnected, no zero sequence current can flow $(\underline{Z}_E \to \infty)$. In zero sequence impedance, the effect of the neutral impedance is trebled, because current $3\underline{I}_0$ is flowing in the neutral conductor.

3.2 Power Calculation in Symmetrical Components

Where quantities are sinusoidal, the complex apparent power in the three-phase system can be calculated from the sum of the three phase powers, as follows:

$$\underline{S} = \underline{U}_1 \underline{I}_1^* + \underline{U}_2 \underline{I}_2^* + \underline{U}_3 \underline{I}_3^* . \tag{3.03}$$

If all voltages and currents are now described by their symmetrical components, the complex apparent power can also be calculated by means of

$$\underline{S} = 3 \cdot \left(\underline{U}_{pos} \underline{I}^*_{pos} + \underline{U}_{neg} \underline{I}^*_{neg} + \underline{U}_0 \underline{I}_0^* \right) \tag{3.04}$$

This relation can be derived from formal calculation on the one hand, and recognized from the equivalent circuit diagrams in Figure 3.1c) on the other hand. Together with the associated currents $\left(\underline{I}_{pos} \underline{I}_{neg} \underline{I}_0\right)$, the voltage sources $(\underline{U}_{pos}, \underline{U}_{neg}, \underline{U}_0)$ tell us the apparent power. Since every equivalent circuit of symmetrical components represents the single-phase equivalent circuit of a symmetrical three-phase system, these powers must be multiplied by a factor of 3.

3.3 Calculating an Asymmetrical Three-Phase System

The symmetrical components method is now demonstrated with the aid of an example. To do so, we will examine the circuit shown in Figure 3.1a). The circuit is supplied by an asymmetrical three-phase system with

$$\underline{U}_1 = 148.7Ve^{j3.6°},\ \underline{U}_2 = 67.5Ve^{-j61.5°} \text{ and } \underline{U}_3 = 148.7Ve^{j165°} \tag{3.05}$$

The associated vector diagram is shown in Figure 3.2a). The self-impedance is $\underline{Z}_L = 10\Omega + j7\Omega$ and the coupling impedance estimated as $\underline{Z}_K = 0\Omega$. The source and load star points should not be connected to one another ($\underline{Z}_E = \infty$).

First of all, transformation equation 3.1 is used to present the source voltages as symmetrical components:

$$\underline{U}_{pos} = 100.4Ve^{j29.3°},\ \underline{U}_{neg} = 49.3Ve^{-j41.3°} \text{ and } \underline{U}_0 = 14.9Ve^{-j16.9°} \tag{3.06}$$

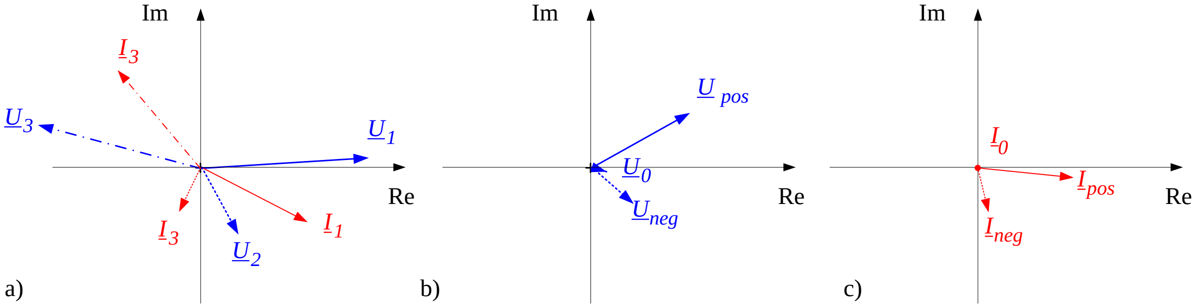

Fig. 3.2: Asymmetrical three-phase system a) Phasor diagram of phase voltages and currents, b) of voltages and c) currents in symmetrical components

The associated vector diagram is shown in Figure 3.2b). In order to fully obtain the elements of the equivalent circuits shown in Figure 3.1, the impedances

$$\underline{Z}_{pos} = \underline{Z}_{neg} = \underline{Z}_L - \underline{Z}_K = 10\Omega + j7\Omega \text{ and } \underline{Z}_0 = \infty \tag{3.07}$$

must be calculated. Therefore, we can now calculate the currents in the symmetrical components.

$$\underline{I}_{pos} = \frac{\underline{U}_{pos}}{\underline{Z}_{pos}} = 8.23Ae^{-j5.7°},\ \underline{I}_{neg} = \frac{\underline{U}_{neg}}{\underline{Z}_{neg}} = 4.04Ae^{-j76.3°} \text{ and } \underline{I}_0 = \frac{\underline{U}_0}{\underline{Z}_0} = 0A \tag{3.08}$$

These currents in symmetrical components can be transformed back into the real system with the aid of equation 3.1.

$$\underline{I}_1 = 10.3Ae^{-j27.4°},\ \underline{I}_2 = 4.32Ae^{-j115.8°} \text{ and } \underline{I}_3 = 11.28Ae^{-j130.1°} \tag{3.09}$$

4. Analysis of Asymmetrical Faults in the Three-Phase Network

If the symmetry of a three-phase system is disrupted, e.g. due to short circuit, ground fault or a power outage, the use of symmetrical components can considerably simplify analysis. This procedure is illustrated by means of the network in Figure 4.1a). The three-phase network is supplied by a symmetrical voltage source ($\underline{E}_1$, $\underline{E}_2$ and $\underline{E}_3$). The internal impedance is cyclic-symmetrical in nature. Under these preconditions, we obtain the equivalent circuits in symmetrical components as shown in Figure 4.1b). In this case, a voltage source is only present in the equivalent circuit diagram of the positive sequence system, with $\underline{E}_{pos} = \underline{E}_1$.

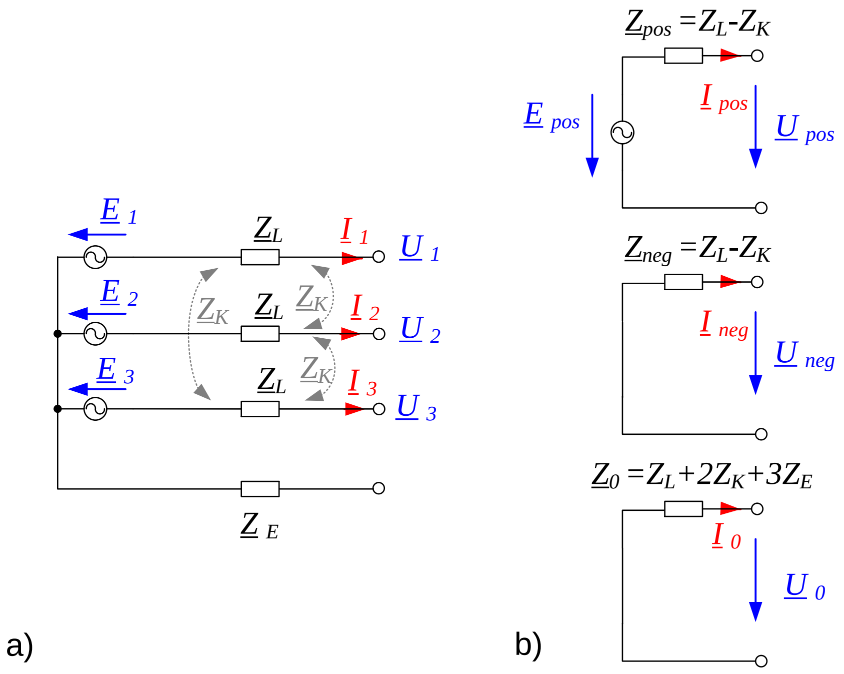

Fig. 4.1: Symmetrical three-phase system with associated equivalent circuit diagrams in symmetrical components

4.1 Two-Pole Short Circuit

The first fault we will examine is a two-pole short circuit without contact with ground. The equivalent circuit diagram in Figure 4.2 shows the asymmetrical fault with a low-impedance connection R between L2 and L3. With a resistor between phases L2 and L3, we see the unbalanced conditions of a real system, as follows:

$$\begin{aligned} \underline{I}_1 &= 0 \\ \underline{I}_2 &= -\underline{I}_3 \\ \underline{U}_2 - \underline{U}_3 &= R \cdot \underline{I}_2 \end{aligned} \tag{4.01}$$

If these equations are transformed into the symmetrical components system, the following relations are obtained:

$$\begin{aligned} \underline{I}_0 &= 0 \\ \underline{I}_{pos} &= -\underline{I}_{neg} \\ \underline{U}_{pos} - \underline{U}_{neg} &= R \cdot \underline{I}_{pos} \end{aligned} \tag{4.02}$$

These relations can be taken into consideration by suitable circuitry in the equivalent circuit diagrams. As shown in Figure 4.2, the positive and negative sequence systems must be connected to the low-ohmic resistor. The zero sequence system is not connected, as no zero sequence system current can flow.

For the currents in the positive and negative sequence systems:

$$\underline{I}_{pos} = -\underline{I}_{neg} = \frac{\underline{E}_{pos}}{\underline{Z}_{pos} + \underline{Z}_{neg} + R} \quad \text{where } \underline{E}_{pos} = \underline{E}_1 \tag{4.03}$$

Back transformation then produces the short-circuit current

$$\underline{I}_2 = -\underline{I}_3 = -j\sqrt{3}\frac{\underline{E}_1}{\underline{Z}_{pos} + \underline{Z}_{neg} + R} \tag{4.04}$$

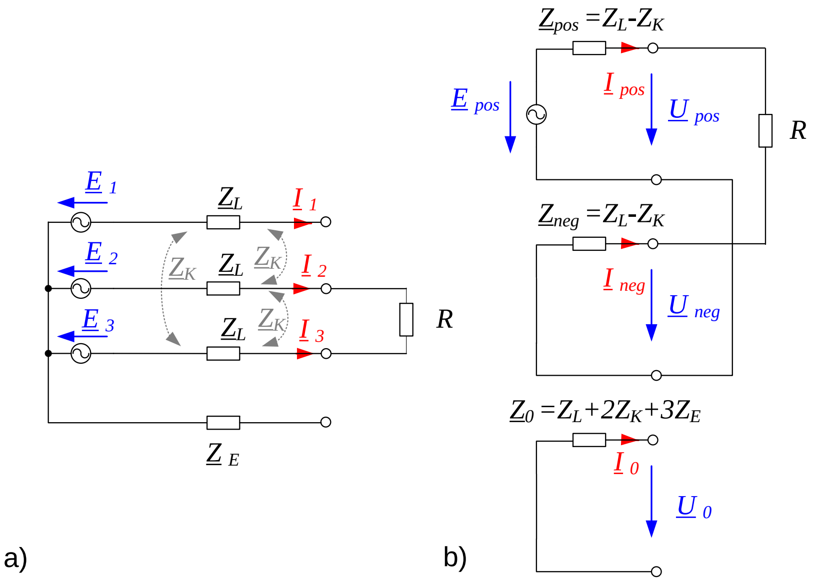

Fig. 4.2: Equivalent circuit for a two-pole short circuit without contact with ground
a) Real three-phase system b) Symmetrical components

4.2 Single-Pole Ground Fault

A low-impedance connection between one of the three conductors (L1, L2, L3) and ground constitutes a single-pole ground fault. In high-voltage networks, the single-pole ground fault is one of the most frequently occurring faults.

To analyze this fault, first of all the real circuit is examined in Figure 4.3b), and the equations that described the fault situation are presented thus

$$\begin{aligned} R\cdot\underline{I}_1 &= \underline{U}_1 \\ \underline{I}_2 &= 0 \\ \underline{I}_3 &= 0 \end{aligned} \tag{4.05}$$

In these equations, the real quantities are now replaced by symmetrical components. Consequently, the fault situation can be described in its entirety using symmetrical components:

$$\begin{aligned} \underline{I}_{pos} &= \underline{I}_{neg} = \underline{I}_0 \\ 3R\cdot\underline{I}_{pos} &= \underline{U}_{pos} + \underline{U}_{neg} + \underline{U}_0 \end{aligned} \tag{4.06}$$

These conditions can be brought about by connecting the equivalent circuit diagrams together, as shown in Figure 4.3b). Since the individual equivalent circuit diagrams are connected in series, all three currents must be identical, as required by equation 4.3. Moreover, the sum of the partial voltage is the same as the voltage drop $3R\cdot\underline{I}_{pos}$. Therefore, the following is obtained for the currents in the positive, negative and zero sequence systems:

$$\underline{I}_{pos} = \underline{I}_{neg} = \underline{I}_0 = \frac{\underline{E}_{pos}}{\underline{Z}_{pos} + \underline{Z}_{neg} + \underline{Z}_E + 3R} \quad \text{where } \underline{E}_{pos} = \underline{E}_1 \tag{4.07}$$

From the equation used for back transformation $\underline{I}_1 = \underline{I}_{pos} + \underline{I}_{neg} + \underline{I}_0$, we obtain the short-circuit current

$$\underline{I}_1 = \frac{\underline{E}_1}{\frac{1}{3}\left(\underline{Z}_{pos} + \underline{Z}_{neg} + \underline{Z}_E\right) + R} \qquad (4.08)$$

for a single-pole ground fault.

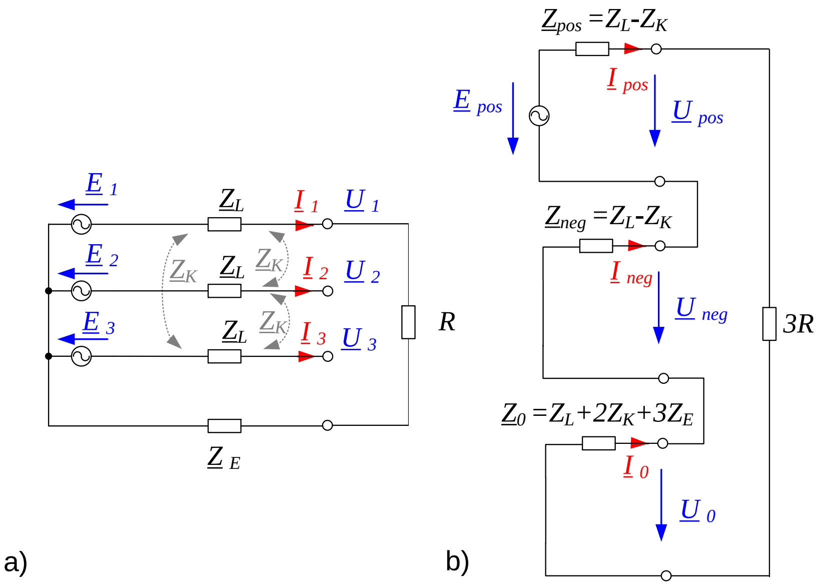

Fig. 4.3: Equivalent circuit for a single-pole ground fault
a) Real three-phase system b) Symmetrical components

5. Measurement of Symmetrical Components

Symmetrical components can be employed to evaluate the behavior of decentralized energy producers in the grid. The characteristics of the grid variables over time are measured at various operating points and incidents, and the symmetrical components of the voltages and currents are measured at the grid connection point. Figure 5.1 illustrates the measurement configuration using a wind turbine as an example. Here, further mechanical variables such as the temperature of the components are also measured, so that the overall system can be evaluated.

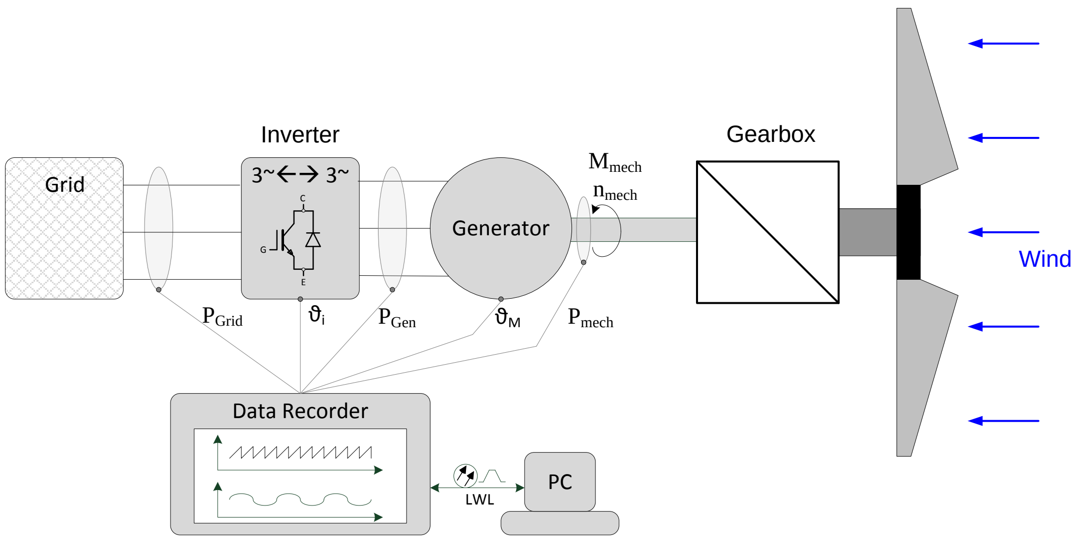

Fig. 5.1: Connection of a data recorder for measuring the characteristic over time of the system variables of a wind turbine

5.1 Phasors

In order to present a real variable as a phasor, first of all the fundamental must be ascertained from the characteristic over time with the aid of a Fourier analysis. This is because in real technical applications in energy systems, pure sine or cosine signals are just a theoretical paradigm. As the following equation shows

$$u_{1f}(2\pi ft) = \hat{u}_{1,\cos} \cdot \cos(2\pi ft) + \hat{u}_{1,\sin} \cdot \sin(2\pi ft) \tag{5.01}$$

the characteristic of the fundamental over time has a sine and a cosine proportion. The amplitudes of these proportions can be obtained using Fourier integrals:

$$\begin{aligned} \hat{u}_{1,\cos} &= \frac{2}{T} \cdot \int_{t-T}^{t} u_1(t)\cos(2\pi ft)\,dt \\ \hat{u}_{1,\sin} &= \frac{2}{T} \cdot \int_{t-T}^{t} u_1(t)\sin(2\pi ft)\,dt \end{aligned} \tag{5.02}$$

Here, T is the cycle duration and $f = 1/T$ the associated frequency of the periodic time characteristic. In electrical engineering, AC quantities are normally presented as complex RMS phasor $\underline{U}_{1f}$.

The relationship between a phasor and the time function is described as follows:

$$u_{1f}(2\pi ft) = \sqrt{2} \cdot \Re\left\{\underline{U}_{1f} e^{j2\pi ft}\right\} = \hat{u}_{1,\cos}\cos(2\pi ft) + \hat{u}_{1,\sin}\sin(2\pi ft) \tag{5.03}$$

By evaluating the equation above, we obtain the real and the imaginary part of phasor as a function of the Fourier coefficients:

$$\underline{U}_{1f} = \frac{\hat{u}_{1,\cos}}{\sqrt{2}} - j\frac{\hat{u}_{1,\sin}}{\sqrt{2}} = U_{1,\cos} - jU_{1,\sin} \tag{5.04}$$

The RMS value of the fundamental can be calculated from the Fourier coefficients by means of

$$U_{1f} = \sqrt{\frac{\hat{u}_{1,\cos}{}^2 + \hat{u}_{1,\sin}{}^2}{2}}\,. \tag{5.05}$$

Figure 5.2 contains an example of this method being used for a voltage and current characteristic. The cycle duration is ascertained at the voltage signal $u_1(t)$. The rectangular function plotted below indicates correct recognition of the cycle duration. The complex RMS value of the current leads the voltage vector by the angle φ.

This mathematical description naturally also applies to voltages and currents in the other two phases

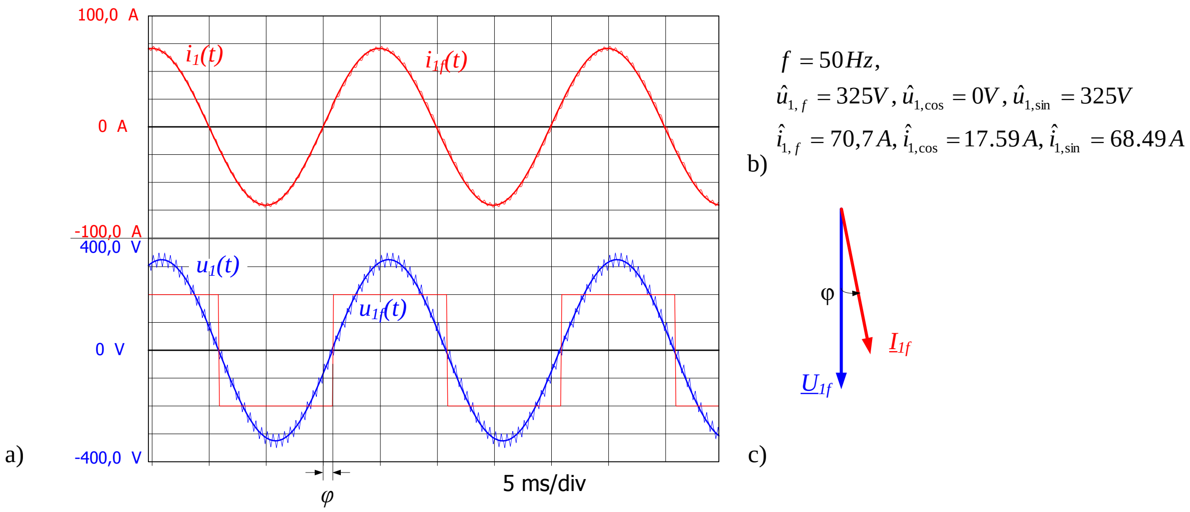

Fig. 5.2: Characteristic of the voltage and current over time, with associated fundamentals and phasors

5.2 Phasors of Symmetrical Components

The defining equations for the symmetrical components give rise to rules of calculation, which enable us to calculate the complex amplitudes of the positive, negative and zero sequence systems from the Fourier coefficients of the measured phase variables. The formulas are obtained as follows, using the positive sequence system as an example.

From the defining equation for the positive sequence component

$$\underline{U}_{pos} = U_{pos,\cos} - jU_{pos,\sin} = \frac{1}{3}\left(\underline{U}_{1f} + \underline{a}\underline{U}_{2f} + \underline{a}^2\underline{U}_{3f}\right) \tag{5.06}$$

$$\text{mit} \quad \underline{a} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \quad \underline{a}^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

and $\underline{U}_{1f} = U_{1,\cos} - jU_{1,\sin}$, $\underline{U}_{2f} = U_{2,\cos} - jU_{2,\sin}$ and $\underline{U}_{3f} = U_{3,\cos} - jU_{3,\sin}$, the complex amplitude of the positive sequence system can be determined by separating them into real and imaginary:

$$\hat{u}_{pos,\cos} = \frac{1}{6}\left(2\hat{u}_{1,\cos} - \hat{u}_{2,\cos} - \hat{u}_{3,\cos} - \sqrt{3}(\hat{u}_{3,\sin} - \hat{u}_{2,\sin})\right)$$
$$\hat{u}_{pos,\sin} = \frac{1}{6}\left(2\hat{u}_{1,\sin} - \hat{u}_{2,\sin} - \hat{u}_{3,\sin} - \sqrt{3}(\hat{u}_{2,\cos} - \hat{u}_{3,\cos})\right) \tag{5.07}$$

The complex amplitude of the negative sequence system can be calculated in the same way.

$$\hat{u}_{neg,\cos} = \frac{1}{6}\left(2\hat{u}_{1,\cos} - \hat{u}_{2,\cos} - \hat{u}_{3,\cos} - \sqrt{3}(\hat{u}_{b,\sin} - \hat{u}_{3,\sin})\right)$$
$$\hat{u}_{neg,\sin} = \frac{1}{6}\left(2\hat{u}_{1,\sin} - \hat{u}_{2,\sin} - \hat{u}_{3,\sin} - \sqrt{3}(\hat{u}_{3,\cos} - \hat{u}_{2,\cos})\right) \tag{5.08}$$

For the zero sequence system, we obtain:

$$u_{0,\cos} = \frac{1}{3\sqrt{2}}\left(\hat{u}_{1,\cos} + \hat{u}_{2,\cos} + \hat{u}_{3,\cos}\right)$$
$$u_{0,\sin} = \frac{1}{3\sqrt{2}}\left(\hat{u}_{1,\sin} + \hat{u}_{2,\sin} + \hat{u}_{3,\sin}\right) \tag{5.09}$$

With the variables of the zero sequence system, it is important to note that RMS values in accordance with Technical Guideline TR3 are used.

The active and reactive power values of the positive sequence system component of the fundamental are:

$$
\begin{aligned}
P_{pos} &= \frac{3}{2}\left(\hat{u}_{pos,\cos}\hat{i}_{pos,\cos} + \hat{u}_{pos,\sin}\hat{i}_{pos,\sin}\right) \\
Q_{pos} &= \frac{3}{2}\left(\hat{u}_{pos,\cos}\hat{i}_{pos,\sin} - \hat{u}_{pos,\sin}\hat{i}_{pos,\sin}\right)
\end{aligned}
\tag{5.10}
$$

In the same way, the active and reactive power values of the negative sequence system component are calculated as follows:

$$
\begin{aligned}
P_{neg} &= \frac{3}{2}\left(\hat{u}_{neg,\cos}\hat{i}_{neg,\cos} + \hat{u}_{neg,\sin}\hat{i}_{neg,\sin}\right) \\
Q_{neg} &= \frac{3}{2}\left(\hat{u}_{neg,\cos}\hat{i}_{neg,\sin} - \hat{u}_{neg,\sin}\hat{i}_{neg,\sin}\right)
\end{aligned}
\tag{5.11}
$$

For the zero sequence system component, the active and reactive power values of the fundamental are:

$$
\begin{aligned}
P_0 &= 3\left(u_{0,\cos}i_{0,\cos} + u_{0,\sin}i_{0,\sin}\right) \\
Q_0 &= 3\left(u_{0,\cos}i_{0,\sin} - u_{0,\sin}i_{0,\sin}\right)
\end{aligned}
\tag{5.12}
$$

A data set of example calculations is available to download [5]. Figure 5.3 presents an abridged version of the results of these calculations. The three-phase current is symmetrical – so here only a positive sequence system is present. The three voltages form an asymmetrical three-phase system. The zero sequence system only exists with the voltages.

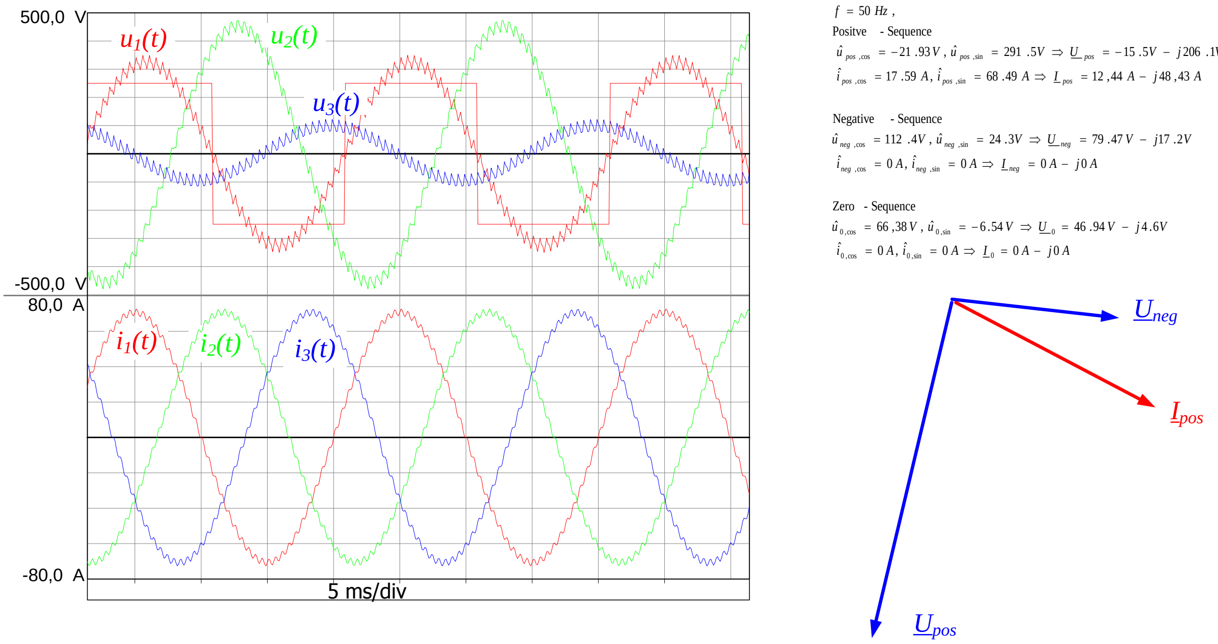

Fig. 5.3: Characteristics of the voltages and currents of a three-phase system over time, with the calculated fundamentals and symmetrical components, plus associated phasor diagram

6. Summary

This article presents the basic principles and use of the symmetrical components method. First of all, we are introduced to the relevant transformation equations. The advantages of using this calculation method are presented using a two-pole and single-pole ground fault as an example. The relationship between the measured characteristics of time-dependent quantities and the associated phasors is discussed at some length. The formulas derived using this approach and the procedure for applying the symmetrical components method can be tested with the aid of a data set.

7. References

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8. Formulas in Perception for Calculating the Symmetrical Components

	Number of Periods	
n	1	
	Allocations	
i_1	@Sqrt(2)*50*@SineWave(1e4; 400e1; 50;0-90) + 2*@SineWave(1e4; 400e1;2000;0)	A
i_2	@Sqrt(2)*50*@SineWave(1e4; 400e1; 50; -120-90)+2*@SineWave(1e4; 400e1;2000;-120)	A
i_3	@Sqrt(2)*50*@SineWave(1e4; 400e1; 50; -240-90)+2*@SineWave(1e4; 400e1;2000;-240)	A
u_1	@Sqrt(2)*230*@SineWave(1e4; 400e1; 50; -15-90)+20*@SineWave(1e4; 400e1;2000;0)+10*@SineWave(1e4;	V
u_2	1.4*@Sqrt(2)*230*@SineWave(1e4; 400e1; 50; -140-90)+ 20*@SineWave(1e4; 400e1;2000;-	V
u_3	0.3*@Sqrt(2)*230*@SineWave(1e4; 400e1; 50; -265-90)+ 20*@SineWave(1e4; 400e1;2000;-	V
	Cycle Information	
cycle_u_1_rfe	@CycleDetect(@FilterBesselLP(Formula.u_1; 1; 100; 1);0; 10)	
cycle_u_1	@CycleDetect((Formula.cycle_u_1_rfe+1)*0,5;0,7;0,1;0)	
t	@Integrate(@Abs(Formula.cycle_u_1))	s
f	@CycleFrequency(Formula.u_1; 1; Formula.cycle_u_1)	Hz
i_1f	@CycleFundamental(Formula.i_1; 1; Formula.cycle_u_1)	A
u_1f	@CycleFundamental(Formula.u_1;1;Formula.cycle_u_1)	V
	Fourier Coeefficients	
u_1_cos	2*@CycleMean(Formula.u_1*@Cosine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	V
u_1_sin	2*@CycleMean(Formula.u_1*@Sine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	V
u_2_cos	2*@CycleMean(Formula.u_2*@Cosine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	V
u_2_sin	2*@CycleMean(Formula.u_2*@Sine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	V
u_3_cos	2*@CycleMean(Formula.u_3*@Cosine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	V
u_3_sin	2*@CycleMean(Formula.u_3*@Sine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	V
i_1_cos	2*@CycleMean(Formula.i_1*@Cosine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	A
i_1_sin	2*@CycleMean(Formula.i_1*@Sine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	A
i_2_cos	2*@CycleMean(Formula.i_2*@Cosine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	A
i_2_sin	2*@CycleMean(Formula.i_2*@Sine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	A
i_3_cos	2*@CycleMean(Formula.i_3*@Cosine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	A
i_3_sin	2*@CycleMean(Formula.i_3*@Sine(2*System.Constants.Pi*Formula.f*Formula.t);1; Formula.cycle_u_1)	A
	Positive Sequence Components	
u_pos_cos	1/6*(2*Formula.u_1_cos-Formula.u_2_cos-Formula.u_3_cos-@Sqrt(3)*(Formula.u_3_sin-Formula.u_2_sin))	V
u_pos_sin	1/6*(2*Formula.u_1_sin-Formula.u_2_sin-Formula.u_3_sin-@Sqrt(3)*(Formula.u_2_cos-Formula.u_3_cos))	V
i_pos_cos	1/6*(2*Formula.i_1_cos-Formula.i_2_cos-Formula.i_3_cos-@Sqrt(3)*(Formula.i_3_sin-Formula.i_2_sin))	A
i_pos_sin	1/6*(2*Formula.i_1_sin-Formula.i_2_sin-Formula.i_3_sin-@Sqrt(3)*(Formula.i_2_cos-Formula.i_3_cos))	A
	Negative Sequence Components	
u_neg_cos	1/6*(2*Formula.u_1_cos-Formula.u_2_cos-Formula.u_3_cos-@Sqrt(3)*(Formula.u_2_sin-Formula.u_3_sin))	V
u_neg_sin	1/6*(2*Formula.u_1_sin-Formula.u_2_sin-Formula.u_3_sin-@Sqrt(3)*(Formula.u_3_cos-Formula.u_2_cos))	V
i_neg_cos	1/6*(2*Formula.i_1_cos-Formula.i_2_cos-Formula.i_3_cos-@Sqrt(3)*(Formula.i_2_sin-Formula.i_3_sin))	A
i_neg_sin	1/6*(2*Formula.i_1_sin-Formula.i_2_sin-Formula.i_3_sin-@Sqrt(3)*(Formula.i_3_cos-Formula.i_2_cos))	A
	Zero Sequence Components	
u_0_cos	1/(3*@Sqrt(2))*(Formula.u_1_cos+Formula.u_2_cos+Formula.u_3_cos)	V
u_0_sin	-1/(3*@Sqrt(2))*(Formula.u_1_sin+Formula.u_2_sin+Formula.u_3_sin)	V
i_0_cos	1/(3*@Sqrt(2))*(Formula.i_1_cos+Formula.i_2_cos+Formula.i_3_cos)	A
i_0_sin	-1/(3*@Sqrt(2))*(Formula.i_1_sin+Formula.i_2_sin+Formula.i_3_sin)	A
	Positive Sequence Power Quantities	
P_pos	3/2*(Formula.u_pos_cos*Formula.i_pos_cos + Formula.u_pos_sin*Formula.i_pos_sin)	W
Q_pos	3/2*(Formula.u_pos_cos*Formula.i_pos_sin - Formula.u_pos_sin*Formula.i_pos_cos)	var
	Negative Sequence Power Quantities	
P_neg	3/2*(Formula.u_neg_cos*Formula.i_neg_cos + Formula.u_neg_sin*Formula.i_neg_sin)	W
Q_neg	3/2*(Formula.u_neg_cos*Formula.i_neg_sin - Formula.u_neg_sin*Formula.i_neg_cos)	var
	Zero Sequence Power Quantities	
P_0	3/2*(Formula.u_0_cos*Formula.i_0_cos + Formula.u_0_sin*Formula.i_0_sin)	W
Q_0	3/2*(Formula.u_0_sin*Formula.i_0_cos - Formula.u_0_cos*Formula.i_0_sin)	var
	Voltage RMS of Positive, Negative and Zero Sequence	
U_pos	@Sqrt(3/2*(Formula.u_pos_cos*Formula.u_pos_cos + Formula.u_pos_sin*Formula.u_pos_sin))	V
U_neg	@Sqrt(3/2*(Formula.u_neg_cos*Formula.u_neg_cos + Formula.u_neg_sin*Formula.u_neg_sin))	V
U_0	@Sqrt(3/2*(Formula.u_0_cos*Formula.u_0_cos + Formula.u_0_sin*Formula.u_0_sin))	V
	Active and Nonactive Current RMS of Positive, Negative and Zero Sequence	
I_P_pos	Formula.P_pos/@Sqrt(3)/Formula.U_pos	A
I_Q_pos	Formula.Q_pos/@Sqrt(3)/Formula.U_pos	A
I_P_neg	Formula.P_neg/@Sqrt(3)/Formula.U_neg	A
I_Q_neg	Formula.Q_neg/@Sqrt(3)/Formula.U_neg	A
I_P_0	Formula.P_0/@Sqrt(3)/Formula.U_0	A
I_Q_0	Formula.Q_0/@Sqrt(3)/Formula.U_0	A
	cos(phi)	
cos_phi_pos	Formula.P_pos/(@Sqrt(Formula.P_pos*Formula.P_pos + Formula.Q_pos*Formula.Q_pos))	
cos_phi_neg	Formula.P_neg/(@Sqrt(Formula.P_neg*Formula.P_neg + Formula.Q_neg*Formula.Q_neg))	
cos_phi_0	Formula.P_0/(@Sqrt(Formula.P_0*Formula.P_0 + Formula.Q_0*Formula.Q_0))	