Measuring the parameters of a transformer equivalent circuit diagram

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HBM Test and Measurement

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Summary

Used in many different applications, the transformer is one of the most important components in alternating current technology. It is used in electrical energy technology to transform between different voltage levels. To ensure efficient transmission of energy in this case, good efficiency and optimum utilization are required. Despite the wide prevalence of power electronic circuits, the transformer is also still required for small power supplies to allow for the required galvanic isolation. It is used in measurement technology to convert measured quantities. Transformers must meet different requirements depending on the intended use. Adaptations to these requirements can be made through the selection of the core material that is used and by varying the geometry of the core. The individual properties of a transformer can be represented by a simple equivalent circuit diagram. This can be used to evaluate how suitable a transformer is for a proposed application and its behavior at various load points.In this article the equivalent circuit diagram of the transformer is first derived and explained. Next measurements and calculation methods for determining the equivalent circuit diagram and the loss of iron in the transformer core are presented. The measurements and calculations are performed with the HBM Genesis 3i data recorder. The appendix contains all the necessary formulas and they can be imported into Perception.

1. Equivalent circuit diagram of the transformer

Figure 1 shows the operating principle of a transformer with two windings that are magnetically connected with a ferrite core. Due to the high permeability of the ferrite core in comparison to the air, the flux Φ_μ is directed through it. Nonetheless, slight leakage fluxes $\Phi_{1\sigma}$ and $\Phi_{2\sigma}$ do occur. Resistances R_1 and R_2 simulate the ohmic part of the windings. To describe the operating behavior of the transformer, an equivalent circuit diagram is derived from this model as shown in Figure 2. This diagram shows the transmission ratio between the primary and secondary side for an ideal transformer. The other effects that occur are represented by passive components. The magnetic fluxes are described by the leakage inductances $L_{1\sigma}$ and $L_{2\sigma}$, and also by the main inductance L_μ. The resistor R_{Fe} is connected in parallel to the main inductance L_μ and serves to simulate the iron losses in the core material. These consist of eddy current losses and hysteresis losses.

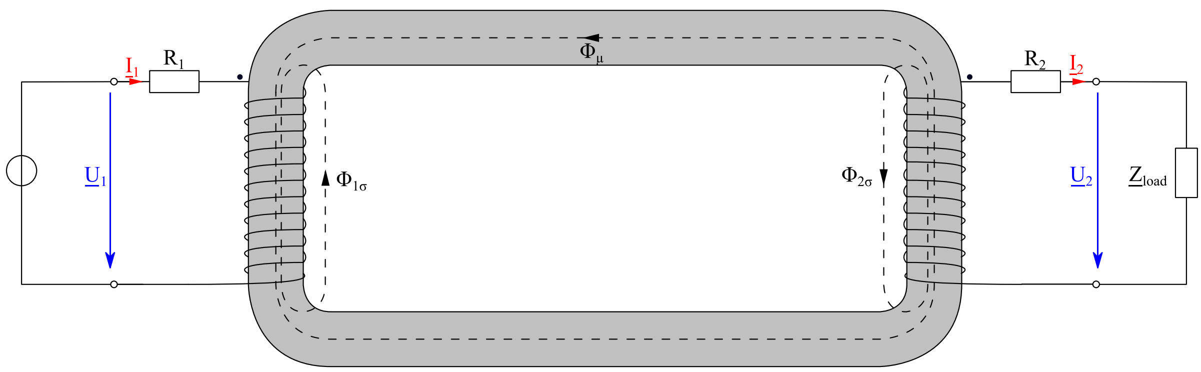


Figure 1: Operating principle of a transformer

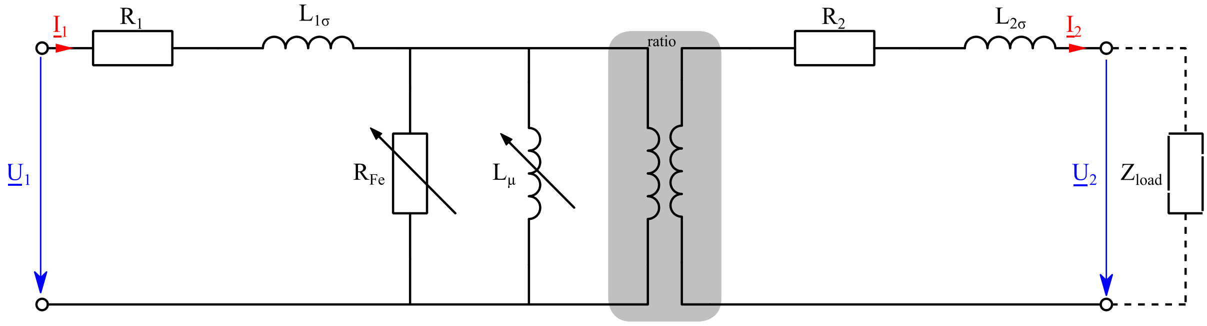


Figure 2: Equivalent circuit diagram of the transformer with ideal transmission

Eddy current losses arise due to a current flow in the ferrite core which is caused by induced voltages. In accordance with Lenz's law, this current opposes the change that caused it. To minimize the current flow, the ferrite core is made up of plates that are isolated from each other. The hysteresis losses are caused by the periodic remagnetization of the ferrite core, since energy is required to align the molecular magnets in the iron (Weiss domains). Since both the main inductance L_μ and the iron loss resistance R_{Fe} are dependent on the core material with non-linear permeability μ_{Fe}, both follow a non-linear course.

$$R_K = R_1 + R_2 \ddot{u}^2 \quad (1)$$

$$L_K = L_{1\sigma} + L_{2\sigma} \ddot{u}^2 \quad (2)$$

The leakage inductances may be considered as linear, since their field lines run primarily through the air, which exhibits constant permeability. For further considerations the equivalent circuit diagram from Figure 2 is simplified still further (Figure 3). The voltage drop on R_1 and $L_{1\sigma}$ is negligibly small in comparison to the voltage drop due to the iron loss resistance R_{Fe} and the main inductance L_μ in normal operation. This makes it possible to contact the iron loss resistance R_{Fe} and main inductance L_μ directly with the input terminals. [1] In equations (1) and (2) the ohmic resistance R_2 and the leakage inductance $L_{2\sigma}$ of the secondary side are converted to the primary side and combined to form R_K and L_K. The measurements and calculations performed below refer to the equivalent circuit diagram simplified in this manner. Quantities $\underline{I'}_2$, $\underline{U'}_2$ and $\underline{Z'}_{load}$ have been converted from the secondary side to the primary side taking into consideration the transmission ratio.

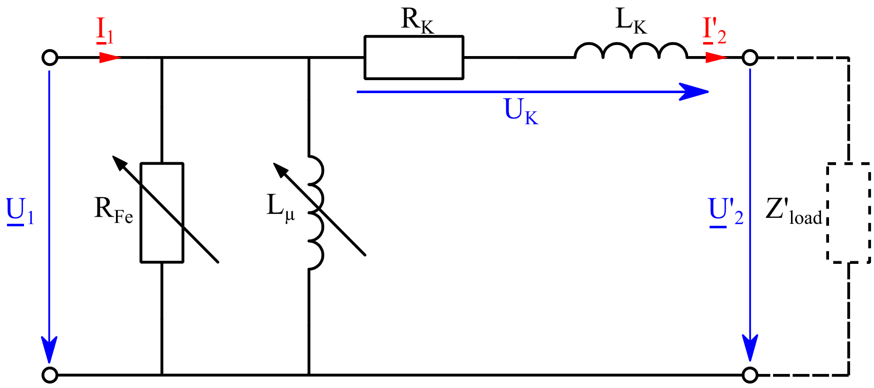


Figure 3: Simplified equivalent circuit diagram of the transformer

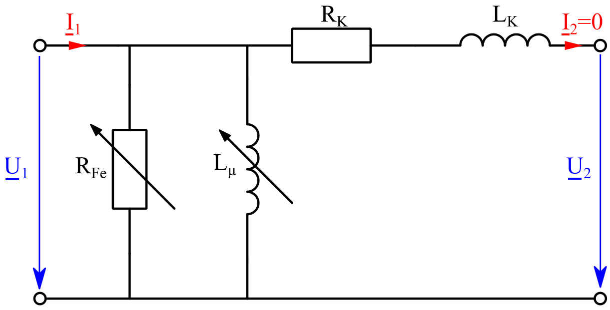


Figure 4: Equivalent circuit diagram of the transformer in no-load

2. Measurements in no-load

The values of the iron loss resistance R_{Fe} and main inductance L_μ can be determined by a no-load test as shown in Figure 4. Since these values exhibit non-linear behavior, the unloaded transformer is supplied with a variable transformer as a sinusoidal voltage source with variable amplitude. This makes it possible to approach and measure different load points with differently linked magnetic flux Ψ. The magnetic flux is calculated from the applied voltage as follows:

$$\Psi = \int \hat{u} \cdot \sin(2\pi f t)\, dt \tag{3}$$

$$\Psi = -\frac{\hat{u}}{2\pi f} \cdot \cos(2\pi f t) \tag{4}$$

The metrologically acquired quantities are the primary voltage u_1 (t), primary current i_1 (t) and secondary voltage u_2 (t). To determine the iron loss resistance R_{Fe} and main inductance L_μ, first the root mean square value of the primary voltage U_1, the primary-side active power P_1 and the reactive power Q_1 are determined. The calculations are performed on a cyclical basis. The component values and transmission ratio ü can be calculated with formulas (5) (6) and (7).

$$R_{Fe} = \frac{U_1^2}{P_1} \tag{5}$$

$$L_\mu = \frac{1}{2\,\pi\,f} \cdot \frac{{U_1}^2}{Q_1} \tag{6}$$

$$ü = \frac{U_1}{U_2} \tag{7}$$

As can be seen in Figure 5, the component values are not constant due to dependence on the magnetic flux. The calculated component values are an average over a sine wave.

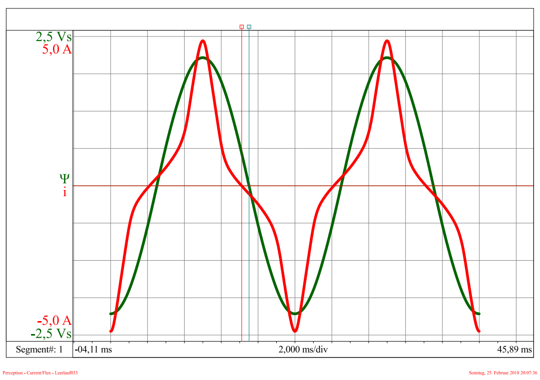


Figure 6: Current and flux

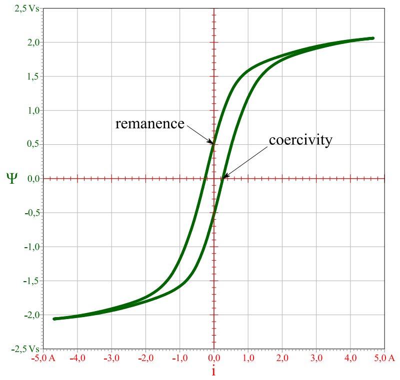


Figure 7: ΨI diagram

The measured values are examined over the course of time to allow for further examination. The distortion of the current over time (red curve) is clearly seen in Figure 6. The core material runs into saturation. The correlation between the flux density B and the magnetic field strength H is illustrated most vividly by the hysteresis curve. If the core geometry is known, the flux density and field strength can be determined from the measured quantities with:

$$B = \frac{\Psi}{A_{\mathrm{Fe}}} \tag{8}$$

$$H = \frac{I}{l_{\mathrm{Fe}}} \tag{9}$$

Due to the unknown core geometry of the test specimen measured here, the hysteresis curve is realized in Figure 7 as a ΨI characteristic curve. The new curve and a number of load points are also shown in Figure 8. The new curve is determined by approaching different load points in which the linked flux and current are acquired in the voltage zero crossing. It is produced when a field strength is first applied to an unmagnetized core and is the characteristic curve of the main inductance L_{μ}. The flux density increases slowly at first. As the field strength increases, the flux density increases faster and faster until the core goes into saturation and the flux density hardly rises at all. Now if the field strength is reduced, the flux density does not return on the new curve. Instead it follows the hysteresis curve. When the field strength is equal to zero a residual magnetism remains, referred to as remanence. The field strength required to eliminate the residual magnetism is called the coercive field strength. [2]

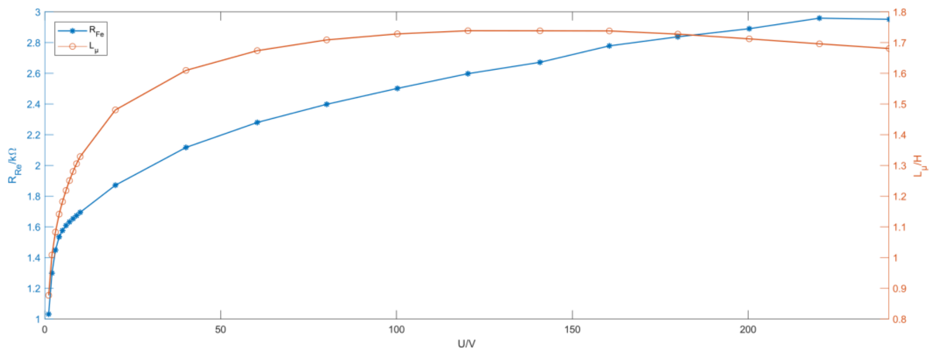


Figure 5: Main inductance and iron loss resistance as a function of the voltage

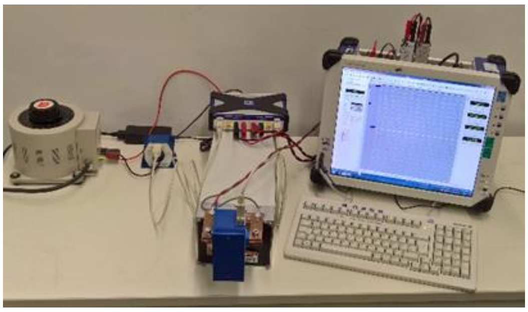
Another method for determining the expected iron losses is the Steinmetz formula (10).

$$P_{Fe} = k \cdot f^{a} \cdot \Psi^{b} \qquad (10)$$

The Steinmetz formula is based on the fact that the surface enclosed by the hysteresis curve is equal to the iron losses. The precondition for applying the Steinmetz formula is a sinusoidal input voltage. The iron losses for differently linked flux calculated from the measured values can be used to determine the unknown coefficients a and b from formula (10) by curve fitting (Figure 9). The curve produced in this manner can then be used to estimate iron losses for other load points in advance.

3. Measurements in the short circuit

In the short-circuit test the secondary side is short-circuited by a low-ohm impedance Z_{load} (Figure 11). The current is set to the nominal (rated) current by a variable transformer. The



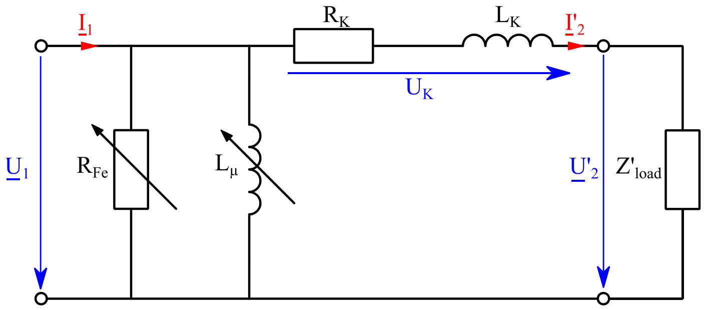


Figure 10: Measurement setup of the short-circuit measurement

Figure 11: Figure 11: Equivalent circuit diagram of the transformer in short circuit

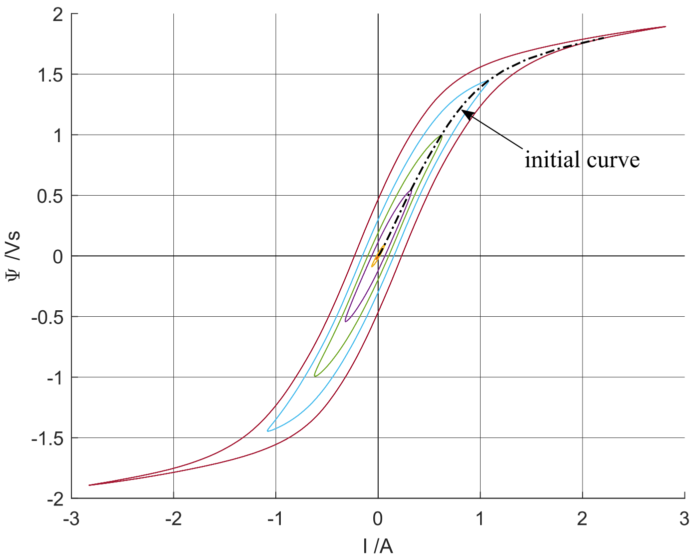


Figure 8: Hysteresis curve with new curve

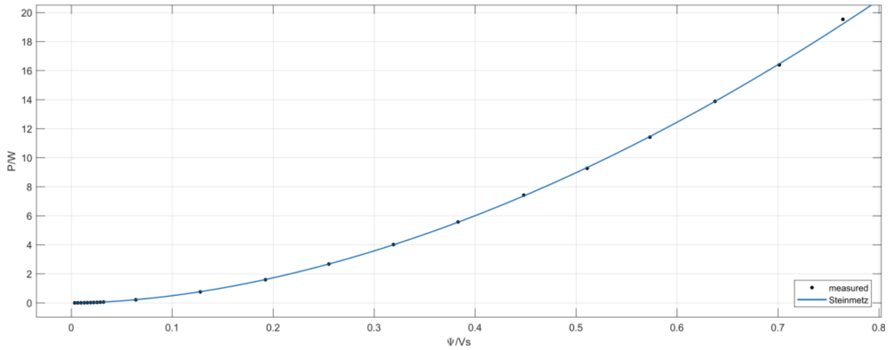


Figure 9: Comparison of iron losses calculated using the Steinmetz formula with the measured data

current through the main inductance and iron loss resistance is negligible in this operating state. The quantities acquired metrologically are the primary voltage u_1 (t), primary current i_1 (t), secondary current i_2 (t) and the voltage u_2 (t) over the load. First the voltage drop is calculated with R_K and L_K.

$$u_K(t) = u_1(t) - \left(\frac{u_2(t)}{ü}\right) \tag{11}$$

Then u_K (t) and i'_2 (t) can be used to calculate the power transformed on R_K and L_K and the component values can be calculated.

$$R_K = \frac{P_K}{{I'_2}^2} \tag{12}$$

$$L_K = \frac{1}{2\pi f} \cdot \frac{Q_K}{{I'_2}^2} \tag{13}$$

4. Sources

[1] J. Teigelkötter, Energieeffiziente elektrische Antriebe, Springer Vieweg Verlag, 2013.

[2] M. S. Hering, Physik für Ingenieure (9.Auflage), Berlin, Heidelberg, New York: Springer, 2004.

Appendix

A1 No-load formulas

Name	Expression	Unit
GEN3i		
	variables allocation	
frequency	50	Hz
	measured value of equivalent circuit	
i_1	Recorder_A.i_1	A
u_1	Recorder_A.u_1	V
u_2	Recorder_A.u_2	V
	#region Cycle Parameters	
	Defining cycle parameters	
Cycle_source	Recorder_A.u_1	V
Cycle_count	1	
Cycle_level	0	V
Cycle_hyst	0,1	V
Cycle_holdoff	0,001	s
Cycle_filter_type	1	
Cycle_cutoff_frequency	1000	Hz
Cycle_direction	0	
Cycle_timeout	1	s
Cycle_source_filt	@HWFilter (RTFormulas.Cycle_source ; RTFormulas.Cycle_filter_type ; RTFormulas.Cycle_cutoff_frequency)	A
	End of cyle parameters	
	#endregion Cycle Parameters	
	==	
	#region Cycle computation and Cycle check	
	START of Computing the CYCLE MASTER	
Cycle_Master	@CycleDetect (RTFormulas.Cycle_source_filt ; RTFormulas.Cycle_count ; RTFormulas.Cycle_level ; RTFormulas.Cycle_hyst ; RTFormulas.Cycle_holdoff ; RTFormulas.Cycle_direction ; RTFormulas.Cycle_timeout)	
	END of Computing the CYCLE MASTER	
	--	
	START computing the CYCLE CHECK (= cycle frequency) to check for missing/multiple cyles	
Cycle_Check	@CycleFrequency (RTFormulas.Cycle_Master)	Hz
	END of Computing the CYCLE CHECK	
	#endregion Cycle computation and Cycle check	
	==	
	START of Computing the True RMS current signals	
I_1	@CycleRMS (RTFormulas.i_1 ; RTFormulas.Cycle_Master)	A
	Computing the mean (or collective) current	
	END of Computing the True RMS current signals	

	START of Computing the True RMS voltage signals	
U_1	@CycleRMS (RTFormulas.u_1; RTFormulas.Cycle_Master)	V
U_2	@CycleRMS (RTFormulas.u_2; RTFormulas.Cycle_Master)	V
	Computing the mean (or collective) voltage	
	END of Computing the True RMS voltage signals	

	START of Computing the electrical power	
	Computing the activ Power	
p_1	RTFormulas.u_1 * RTFormulas.i_1	W
P_1	@CycleMean (RTFormulas.p_1 ; RTFormulas.Cycle_Master)	W
	Computing the apparent power	
S_1	RTFormulas.U_1 * RTFormulas.I_1	VA
	Computing the reactive Power	
Q_1	@Sqrt(RTFormulas.S_1*RTFormulas.S_1-RTFormulas.P_1*RTFormulas.P_1)	var
	--	
	#endregion Power computations	
	===	
	#endregion AC3_PN	
	***	
R_Fe	(RTFormulas.U_1*RTFormulas.U_1)/RTFormulas.P_1	Ω
L_µ	((RTFormulas.U_1*RTFormulas.U_1)/RTFormulas.Q_1)/(2*System.Constants.Pi*RTFormulas.frequency)	H
	Computing Ψ	
Ψ	@Integrate(RTFormulas.u_1)	Vs

A2 Formeln Kurzschluss

Name	Expression	Unit
GEN3i		
	variables allocation	
ratio	40	
frequency	50	Hz
	measured value of equivalent circuit	
i_1	Recorder_A.i_1	A
i'_2	Recorder_A.i_2/RTFormulas.ratio	A
u_1	Recorder_A.u_1	V
u'_2	Recorder_A.u_2*RTFormulas.ratio	V
u_K	Recorder_A.u_1-RTFormulas.u'_2	V
	#region Cycle Parameters	
	Defining cycle parameters	
Cycle_source	Recorder_A.u_1	V
Cycle_count	1	
Cycle_level	0	V
Cycle_hyst	0,1	V

Cycle_holdoff	0,001	s
Cycle_filter_type	1	
Cycle_cutoff_frequency	1000	Hz
Cycle_direction	0	
Cycle_timeout	1	s
Cycle_source_filt	@HWFilter (RTFormulas.Cycle_source ; RTFormulas.Cycle_filter_type ; RTFormulas.Cycle_cutoff_frequency)	A
	End of cyle parameters	
	#endregion Cycle Parameters	
	==	
	#region Cycle computation and Cycle check	
	START of Computing the CYCLE MASTER	
Cycle_Master	@CycleDetect (RTFormulas.Cycle_source_filt ; RTFormulas.Cycle_count ; RTFormulas.Cycle_level ; RTFormulas.Cycle_hyst ; RTFormulas.Cycle_holdoff ; RTFormulas.Cycle_direction ; RTFormulas.Cycle_timeout)	
	END of Computing the CYCLE MASTER	
	--	
	START computing the CYCLE CHECK (= cycle frequency) to check for missing/multiple cyles	
Cycle_Check	@CycleFrequency (RTFormulas.Cycle_Master)	Hz
	END of Computing the CYCLE CHECK	
	#endregion Cycle computation and Cycle check	
	==	
	START of Computing the True RMS current signals	
I_1	@CycleRMS (RTFormulas.i_1 ; RTFormulas.Cycle_Master)	A
I'_2	@CycleRMS(RTFormulas.i'_2;RTFormulas.Cycle_Master)	A
	Computing the mean (or collective) current	
	END of Computing the True RMS current signals	
	--	
	START of Computing the True RMS voltage signals	
U_1	@CycleRMS (RTFormulas.u_1; RTFormulas.Cycle_Master)	V
U'_2	@CycleRMS (RTFormulas.u'_2; RTFormulas.Cycle_Master)	V
U_K	@CycleRMS (RTFormulas.u_K; RTFormulas.Cycle_Master)	V
	Computing the mean (or collective) voltage	
	END of Computing the True RMS voltage signals	
	--	
	START of Computing the electrical power	
	Computing the activ Power	
p_in	RTFormulas.u_1 * RTFormulas.i_1	W
p_load	RTFormulas.u'_2*RTFormulas.i'_2	W
p_K	RTFormulas.u_K*RTFormulas.i'_2	W
P_in	@CycleMean (RTFormulas.p_in ; RTFormulas.Cycle_Master)	W
P_load	@CycleMean(RTFormulas.p_load;RTFormulas.Cycle_Master)	W
P_K	@CycleMean(RTFormulas.p_K;RTFormulas.Cycle_Master)	W
	Computing the apparent power	

S_in	RTFormulas.U_1 * RTFormulas.I_1	VA
S_load	RTFormulas.U'_2 * RTFormulas.I'_2	VA
S_K	RTFormulas.U_K * RTFormulas.I_1	VA
	Computing the reactive Power	
Q_in	@Sqrt(RTFormulas.S_in*RTFormulas.S_in-RTFormulas.P_in*RTFormulas.P_in)	var
Q_load	@Sqrt(RTFormulas.S_load*RTFormulas.S_load-RTFormulas.P_load*RTFormulas.P_load)	var
Q_K	@Sqrt(RTFormulas.S_K*RTFormulas.S_K-RTFormulas.P_K*RTFormulas.P_K)	var
	--	
	#endregion Power computations	
	==	
	Computing the Parameter of the short circuit	
R_K	RTFormulas.P_K/(RTFormulas.I_1*RTFormulas.I_1)	Ω
L_K	(RTFormulas.Q_K/(RTFormulas.I_1*RTFormulas.I_1))/(2*System.Constants.Pi*RTFormulas.frequency)	H
	Computing the Parameter of load	
R_load	RTFormulas.P_load/(RTFormulas.I'_2*RTFormulas.I'_2)	Ω
L_load	(RTFormulas.Q_load/(RTFormulas.I'_2*RTFormulas.I'_2))/(2*System.Constants.Pi*RTFormulas.frequency)	H