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Linearity and Sensitivity Error in the Use of Single Strain Gages with Voltage-Fed and Current-Fed Circuits

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An important part of experimental stress analysis is the measurement of strain with single strain gages and the type of circuitry employed influences the linearity and sensitivity of the measurements. This article describes the basic principles of strain measurement by means of strain gages, from the strain itself via the variation in resistance to the eventual electrical signal, and compares the magnitude of error of the voltage-fed bridge circuit and the current-fed circuit. The conclusion reached is that, for single strain gages, voltage-fed bridge circuits give better linearity by several powers of ten and, when there is initial detuning, a similarly more stable sensitivity than current-fed circuits.

1. Introduction

Current-fed circuits possess a very linear characteristic for the measurement of resistance and changes in resistance. However, this does not apply to the measurement of strain by means of single strain gages. In such applications the current-fed circuit exhibits a much greater linearity and sensitivity error than the voltage-fed Wheatstone bridge circuit. **Fig. 1** shows the circuit diagrams of the voltage-fed and current-fed circuits for single strain gages. The cause of the different linearity and sensitivity characteristics is the not precisely linear relationship between the strain and the relative change in resistance of the strain gage. In the case of the voltage-fed Wheatstone bridge circuit this non-linearity is opposite to the non-linearity of the circuit, with the result that the two tend to cancel each other out.

In order to explain this self-compensation process in more detail, it is necessary to examine the relationship between the strain and the electrical signal produced. **Fig. 2** shows the various stages of the process from measurement of the actual strain to the eventual production of an electrical signal. The strain in the specimen causes a relative change in length of the strain gage which, in turn, causes a relative change in resistance which is converted into an electrical signal in the relevant circuit. The relationships between the various parameters in the individual steps of the process will be described individually later and then the relationships between the strain and the electrical signals from the voltage-fed quarter bridge circuit and current-fed circuit derived.
In the following section it will be explained why the actual, i.e. effective, strain ε_e is only approximately equal to the relative change in length $\Delta l/l_o$.

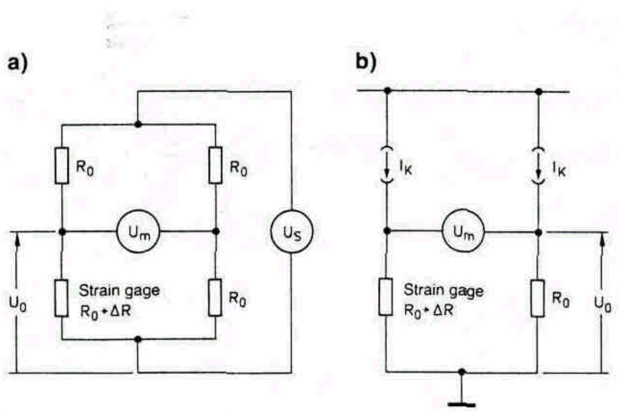

Fig. 1: Circuit diagrams for single strain gages
a) voltage-fed bridge circuit
b) current-fed circuit

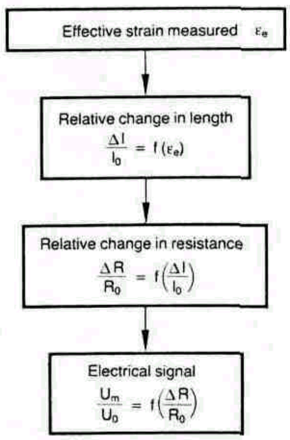

Fig. 2: The steps from measurement of the strain to the eventual electrical signal

2. The definition of strain

According to VDI/VDE 2635 [1], technical strain e is the ratio of elongation or contraction Al to the original length l_o

$$\varepsilon = \frac{\Delta l}{l_o}. \tag{1}$$

ΔThis definition leads to difficulties in the addition and subtraction of large values of strain, and this will be demonstrated by means of an example: When a wire 1000 mm long is stretched by 10,000 µm/m it means that its length increases by 10 mm to 1010 mm. If the same wire is stretched by 10,000 (µm/m again in a second, unconnected measurement, this time its length increases by 10.1 mm because the original length for the second measurement was 1010 mm. Therefore, the total elongation is 20.1 mm. If the original wire had been stretched by 20,000 µm/m for the first measurement it would only have increased in length by 20 mm according to the definition given in equation (1).

Consequently, the two independent stretchings of 10,000 µm/m each do not correspond to a single stretching of 20,000 µm/m but to one of 20,100 µm/m, with the first accounting for 10,000 µm/m and the second for 10,100 µm/m. Naturally, this is a totally unsatisfactory situation because it means that each measurement is dependent on the prior measurement taken with the device. It is essential, of course, for the same values of strain to give rise to the same readings and this can only be assured by restricting the definition of strain e in equation (1) to very small values of e and, therefore, Δl.

Equation (1) becomes totally correct if Δl is made infinitely small in relation to dl and divided by the total length l:

$$d\varepsilon = \frac{dl}{l}. \tag{2}$$

The precise equation for de is

$$d\varepsilon + d\varepsilon = 2d\varepsilon. \tag{3}$$

Every value of effective strain ee can be regarded as the sum of an infinite number of infinitely small values of strain:

$$\varepsilon_e = \int d\varepsilon. \tag{4}$$

Together with equation (2) this gives

$$\varepsilon_e = \int \frac{dl}{l}. \tag{5}$$

Equation (5) describes the "logarithmic" or effective strain which has been known in this form since 1909 [2].

When a wire, e.g. a strain gage, of length l_0 is stretched to length $l_0 + \Delta l$, the effective strain ε_e becomes

$$\varepsilon_e = \int_{l_o}^{l_o+\Delta l} \frac{dl}{l} = \ln\left(1 + \frac{\Delta l}{l_o}\right). \tag{6}$$

Equation (6) represents an exact definition for strains of any magnitude. According to this equation, measurements of strain are independent of the prior history and part-strains can be added or subtracted without incurring the discrepancies described in the previous example.

Comparing the effective strain ε_e with the strain ε defined in equation (1) gives the following relationships:

$$\varepsilon_e = \ln(1 + \varepsilon); \tag{7}$$

$$\varepsilon = \exp(\varepsilon_e) - 1. \tag{8}$$

For small values of strain $\varepsilon \approx \varepsilon_e$ as a first approximation, which is apparent if equation (8) is written as a series:

$$\varepsilon = \varepsilon_e + \frac{1}{2}\varepsilon_e^2 + \frac{1}{6}\varepsilon_e^3 + \ldots \tag{9}$$

Therefore, for small values of strain both definitions give the same results. The relation between ε_e and ($\varepsilon - \varepsilon_e$) is shown in **Fig. 3.**

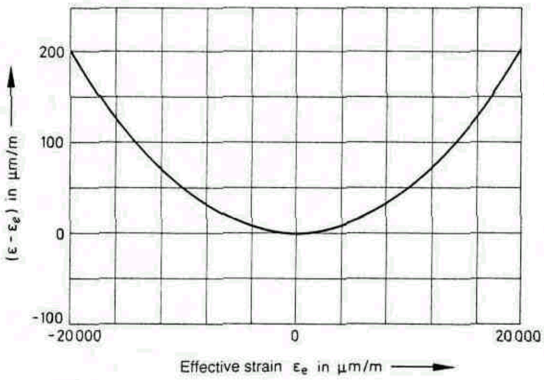

Fig. 3: Deviation of strain e from the effective straingе in relation to the effective strain

3. Theoretical relationship between the relative change in length and the relative change in resistance of strain gages

The following observations are only valid for strain gages having metal grids and a k factor of approximately 2. However, since these are the most popular types of strain gage the restriction is of no importance in the majority of practical applications. The definition of k factor is

$$\frac{\Delta R}{R_o} = k\varepsilon \tag{10}$$

Therefore, k = 2 means that the relative change in resistance is twice the strain e. The reason for specifying strain gages with a k factor of 2 is because the k factor is always 2 in the plastic range of every material [3].

For metal strain gages with a k factor of 2, change in form accounts for approximately 80% of the k factor in the elastic range and a change in specific resistance for only about 20%.

In the plastic range of a material the relative change in resistance is caused exclusively by the change in form with simultaneous constancy of volume and specific resistance. Therefore, strain gages with a k factor of 2 possess constant sensitivity in both the elastic and plastic ranges, which is a very important requirement when measuring large values of strain, in order to obtain reliable readings. Another advantage is that the readings are therefore also almost entirely independent of the prior treatment and use of the strain gage.
The resistance of a conductor is calculated with the formula

$$R = \frac{\varrho \cdot l}{A}, \qquad (11)$$

where l is the length, A the cross section and ϱ the specific resistance of the conductor. If the volume V and specific resistance ϱ are assumed to remain constant (plastic strain), the following relationship between the resistance and the length can be derived:

$$V = l \cdot A = \text{const.} \qquad (12)$$

giving

$$A \sim \frac{1}{l} \qquad (13)$$

and

$$R \sim l^2 \qquad (14)$$

$$R = R_o + \Delta R \qquad (15)$$

$$(R_o + \Delta R) \sim (l_o + \Delta l)^2 \qquad (16)$$

$$\Delta R \sim 2\Delta l \cdot l_o + \Delta l^2. \qquad (17)$$

This gives the relative change in resistance

$$\frac{\Delta R}{R_o} = \frac{2\Delta l}{l_o} + \left(\frac{\Delta l}{l_o}\right)^2. \qquad (18)$$

With equation (1) this becomes

$$\frac{\Delta R}{R_o} = 2\varepsilon + \varepsilon^2 \qquad (19)$$

and, as a first approximation,

$$\frac{\Delta R}{R_o} \approx 2\varepsilon. \qquad (20)$$

Therefore, the derivation gives a k factor of 2 as a first approximation. When considering the linearity, however, the term ε^2 in equation (19) must not be neglected.

4. Relationship between measured voltage and change in resistance

When a strain gage is connected as shown in Fig. 1 a with three equal resistors R_0 in a Wheatstone bridge, and the resistance of the strain gage changes from R_0 to $R_0 + \Delta R$ due to the strain e, the measured voltage U_m can be calculated from

$$U_m = \frac{U_S}{2} \frac{\frac{\Delta R}{R_o}}{2 + \frac{\Delta R}{R_o}} \qquad (21)$$

The voltage $U_S/2$ is that across the resistors R_0 and, therefore, also across the strain gage when it is unstrained; for the purposes of this description it will be denoted by U_0:

$$U_o = \frac{U_S}{2}. \qquad (22)$$

It is normally necessary to know the magnitude of the measured voltage U_m (change in voltage) in relation to the voltage U_0 across the strain gage when it is unstressed. Dividing the two voltages gives

$$\frac{U_m}{U_o} = \frac{\frac{\Delta R}{R_o}}{2 + \frac{\Delta R}{R_o}} \qquad (23)$$

the formula for the voltage-fed bridge circuit. The relationship between measured signal and change in resistance is non-linear.
In the case of the constant-current circuit as in Fig. 1b is

$$U_o = I_K R_o \text{ and } U_m = I_K \Delta R. \qquad (24)$$

Which gives the formula for the constant-current circuit

$$\frac{U_m}{U_o} = \frac{\Delta R}{R_o}. \qquad (25)$$

Therefore, the measured signal varies proportionally with ΔR.

5. Relationship between the effective strain and the electrical signal

The next step is to determine the overall transmissibility of the measured value "effective strain ε_e " to the electrical measured signal "U_m/U_0" for voltage-fed bridge circuits and current-fed circuits. Substituting equation (8) in equation (19) gives

$$\frac{\Delta R}{R_o} = \exp(2\varepsilon_e) - 1. \qquad (26)$$

For metal strain gages with a k factor of 2, equation (26) represents the theoretical relationship between the effective strain ε_e and the relative change in resistance R/R_0. The series expansion of equation (26) gives

$$\frac{\Delta R}{R_o} \approx 2(\varepsilon_e + \varepsilon_e^2) \qquad (27)$$

which is a good approximate solution. Practical measurements have given the same result in the past [4].
In order to obtain the relationship between the effective strain ee and the electrical signal U_m/U_0 for voltage-fed bridge circuits, equation (26) is substituted in equation (23), giving

$$\frac{U_m}{U_o} = \frac{\exp(2\varepsilon_e) - 1}{\exp(2\varepsilon_e) + 1}. \qquad (28)$$

Multiplying the numerator and denominator of equation (28) by $\exp(-\varepsilon_e)$ gives

$$\frac{U_m}{U_o} = \tanh(\varepsilon_e). \qquad (29)$$

Therefore, the theoretical relationship between the electrical signal U_m/U_0 and the effective strain ε_e for an ideal metal strain gage with a k factor of 2 is determined by the hyperbolic tangent.

The third approximation of equation (29) is given by

$$\frac{U_m}{U_o} \approx \varepsilon_e - \frac{1}{3}\varepsilon_e^3 \qquad (30)$$

The error of this approximation compared to the exact solution is extremely slight; even with a very large strain of 100,000 µm/m it is only - 0.0013% of the actual value.
From equation (30) it can be seen that the measured signal increases linearly with the strain at small values of strain. Only with very large values of strain does the term $\varepsilon^3_e/3$ produce a measurable linearity error. Even then, it is only 0.3 µm/m for a strain of 10,000 µm/m. **Fig. 4** shows the linearity error as a function of the effective strain ee for the voltage-fed bridge circuit.

In the case of current-fed circuits the electrical signal is equal to the relative change in resistance of the strain gage so that, by substituting equation (26) in equation (25), the relationship between the effective strain and the electrical signal is obtained directly:

$$\frac{U_m}{U_o} = \exp(2\varepsilon_e) - 1. \qquad (31)$$

Expanding the series to the third term gives the third approximation of equation (31):

$$\frac{U_m}{U_o} \approx 2\left(\varepsilon_e + \varepsilon_e^2 + \frac{2}{3}\varepsilon_e^3\right). \qquad (32)$$

For an effective strain of 10,000 µm/m the current-fed circuit has a linearity error of 100.7 µm/m. **Fig.** 5 shows the linearity error in relation to the effective strain for the current-fed circuit.

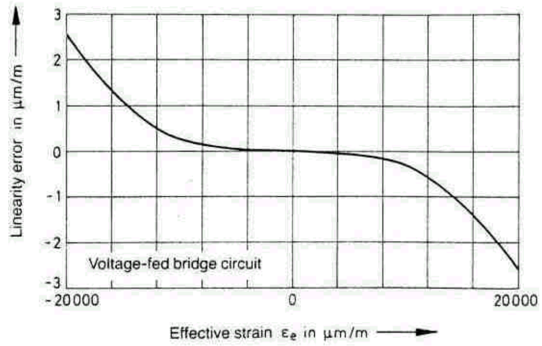

Fig. 4: Small linearity error of the voltage-fed bridge circuit in relation to the effective strain

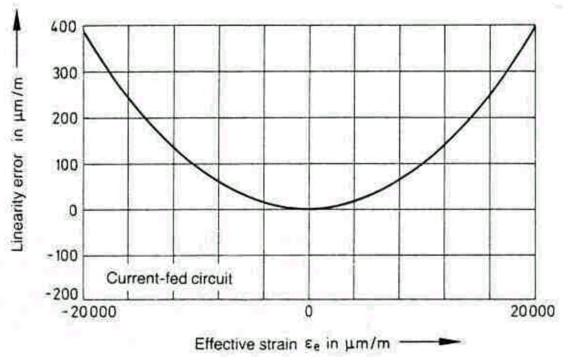

Fig. 5: Large linearity error of the current-fed bridge circuit in relation to the effective strain

6. The effect of extreme initial detuning on measuring sensitivity

So far, this article has dealt mainly with the linearity error of voltage-fed bridge circuits and current-fed circuits at very large values of strain. However, for most strain measurements, errors in linearity are not of overwhelming interest because the strains to be measured are often relatively small and the linearity error can be neglected within the range of accuracy required. Nevertheless, the relationships which have been derived are of considerable significance when considering the effect on measuring sensitivity of prestrain and the resistance tolerances of the strain gages, i.e. initial detuning in general.
The measuring sensitivity E for very small strains is given by the slope of the transmission curve at the particular working point, i.e. at the particular value of effective strain ε_e:

$$E = \frac{d\left(\frac{U_m}{U_o}\right)}{d\varepsilon_e} \qquad (33)$$

The values of valid, effective strain ee are obtained by converting the resistance errors $\Delta R/R_0$ due to the resistance tolerances and prestrain of the strain gages according to equation (34);

$$\varepsilon_e = \frac{1}{2} \ln \left(\frac{\Delta R}{R_o} + 1 \right). \qquad (34)$$

The sensitivity E_B of the voltage-fed bridge circuit is then

$$E_B = \frac{d[\tanh(\varepsilon_e)]}{d\varepsilon_e} \qquad (35)$$

and therefore

$$E_B = 1 - [\tanh(\varepsilon_e)]^2. \qquad (36)$$

Equation (36) can be written in the following approximate form:

$$E_B \approx 1 - \varepsilon_e^2 + \frac{2}{3} \varepsilon_e^4. \qquad (37)$$

The sensitivity E_S of current-fed circuit can be calculated similarly:

$$E_S = 2\exp(2\varepsilon_e) \qquad (38)$$

and written in the approximate form:

$$E_S \approx 2(1 + 2\varepsilon_e + 2\varepsilon_e^2 + \frac{4}{3} \varepsilon_e^3). \qquad (39)$$

A point of interest is how the measuring sensitivity E varies in relation to the effective strain ε_e. If the sensitivity for ε_e =0 is denoted E_0 and the sensitivity at the appropriate working point on the curve E, the relative change in sensitivity $\Delta E/E_0$ can be written:

$$\frac{\Delta E}{E_o} = \frac{E - E_o}{E_o}. \qquad (40)$$

Substituting the values from equation (36) for the voltage-fed bridge circuit in equation (40) gives the relative change in sensitivity:

$$\frac{\Delta E_B}{E_{Bo}} = - (\tanh \varepsilon_e)^2 \qquad (41)$$

which can be written approximately:

$$\frac{\Delta E_B}{E_{Bo}} \approx - \varepsilon_e^2 + \frac{2}{3} \varepsilon_e^4. \qquad (42)$$

Treating equation (40) for the current-fed circuit similarly with equation (38) gives the relative change in sensitivity of the current-fed circuit:

$$\frac{\Delta E_S}{E_{So}} = \exp(2\varepsilon_e) - 1 \qquad (43)$$

which can be written approximately:

$$\frac{\Delta E_S}{E_{So}} \approx 2\varepsilon_e + 2\varepsilon_e^2 + \frac{4}{3} \varepsilon_e^3. \qquad (44)$$

Fig. 6 shows the relative variation in sensitivity with effective strain for the voltage-fed bridge circuit and Fig. 7 the same for the current-fed circuit. The relative variation in sensitivity of voltage-fed bridge circuits in the range from - 20,000 (µm/m ≦ ε_e ≦ 20.000 µm is less than 0.04% whereas that of the current-fed circuit in the same range is approximately ±4%.

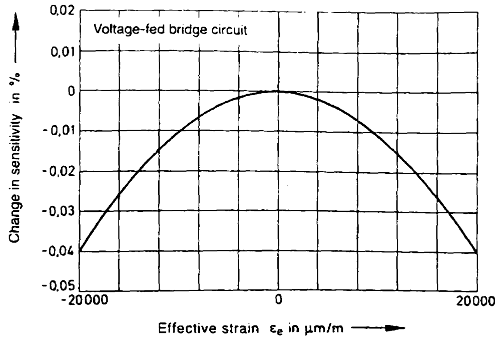

Fig. 6: Small variation in sensitivity in relation to effective strain for the voltage-fed bridge circuit

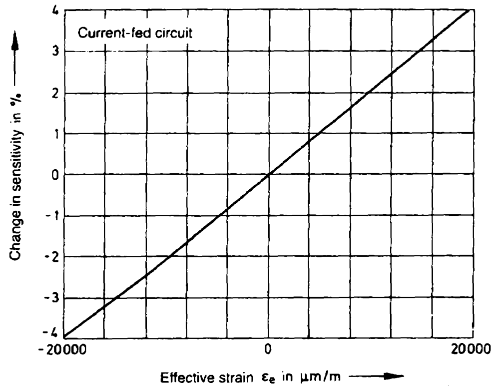

Fig. 7: Large variation in sensitivity in relation to effective strain for the current-fed bridge circuit

Substituting equation (34) in equations (41) and (43) gives the relative change in sensitivity in relation to the relative change in resistance $\Delta R/R_0$ of the active strain gage.

For voltage-fed circuits the following equation can be written:

$$\frac{\Delta E_B}{E_{Bo}} = - \left(\frac{\frac{\Delta R}{R_o}}{2 + \frac{\Delta R}{R_o}} \right)^2 \qquad (45)$$

and in the approximate form:

$$\frac{\Delta E_B}{E_{Bo}} \approx - \frac{1}{4} \left(\frac{\Delta R}{R_o} \right)^2. \qquad (46)$$

For current-fed circuits the equation is:

$$\frac{\Delta E_S}{E_{So}} = \frac{\Delta R}{R_o}. \qquad (47)$$

Figs. 8 and 9 show the relative variation in sensitivity in relation to the relative change in resistance $\Delta R/R_0$ of the active strain gage. It can be seen that, with voltage-fed bridge circuits, relative changes in resistance of ± 6% cause changes in measuring sensitivity of less than 0.1%.

In the case of the current-fed circuit the variation in measuring sensitivity is proportional to the variation in resistance and is 6% for a resistance of 6%. This disadvantage of the current-fed circuit has already been referred to in [5].

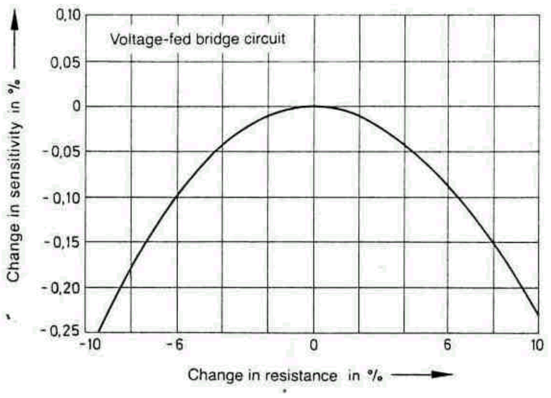

Fig. 8: Small variation in sensitivity in relation to change in resistance of the strain gage for a voltage-fed bridge circuit

As a result of this fact it is necessary to match the constant current for the strain gage to the actual value of strain gage resistance if the sensitivity error of the current-fed circuit is to be eliminated. It means that zero balancing of the hardware must be employed, which involves a considerable amount of additional circuitry in the case of multiple-gage systems.

With voltage-fed bridge circuits, on the other hand, a simple computing zero balance is sufficient up to very large values of initial detuning.

their effect cannot be calculated either for the voltage-fed circuit or the current-fed circuit and this is the reason for comparing the linearity and sensitivity error of the two types of circuit on the basis of systematic, calculable relationships.

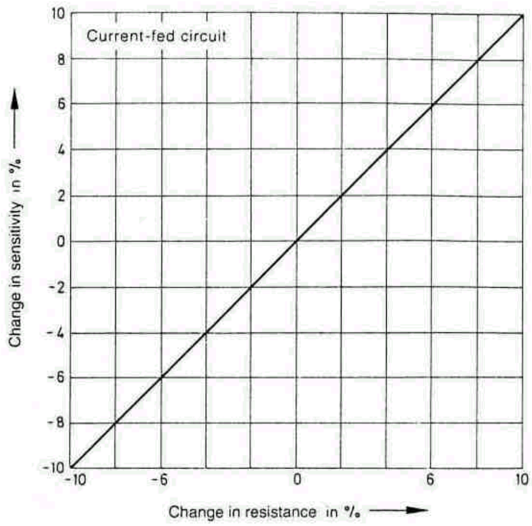

Fig. 9: Large variation in sensitivity in relation to change in resistance of the strain gage for a current-fed bridge circuit

The general conclusion can be drawn that, for the use of strain gages in quarter bridge connection, the voltage-fed bridge circuit is superior to the current-fed circuit in many ways, primarily because of better linearity by several powers of ten and similarly better sensitivity stability under conditions of initial detuning.

7. Summary

The purpose of this article is to draw attention to the fact that the linearity and sensitivity of strain gage measurements cannot be assessed solely from the relation between change in resistance and change in electrical signal; the transmissibility between strain and change in resistance must also be taken into account. However, one important proviso to the relationships developed in the article is that they are only precisely valid for ideal strain gages applied ideally. In reality the transmissibility of "strain/change in resistance" is adversely affected by many factors such as less-than-ideal backing materials and adhesives and multiple-axial stress fields in the grid of the strain gages. Such factors are usually unsystematic and, therefore, cannot be dealt with in a universally-applicable form. Moreover,

8. References

[1] VDI/VDE 2653 Blatt 1: Dehnungsmeßstreifen mit metallischem Meßgitter, Kenngrößen und Prüfbedingungen : August 1974

[2] Ludwik, P.: Elemente der Technologischen Mechanik; Verlag von Julius Springer. Berlin 1909

[3] Rohrbach, Chr.: Handbuch für elektrisches Messen mechanischer Größen (pp. 126 and 130); VDI-Verlag GmbH, Düsseldorf 1967

[4] Hoffmann, K.: Zum Linearitätsfehler bei Dehnungsmessungen mit Dehnungsmeßstreifen im Hochdehnungsbereich; MTB 12 (1976), pp. 53-57

[5] Knapp, H.: Vorschläge zur Vereinfachung und Verbesserung der Schaltungstechnik bei rechnergesteuerten Vielstellen-Dehnungsmeßgeräten;
tm 1981, pp. 265-271