Introduction to Structural Dynamics

MEASUREMENTS AND ANALYSIS

Niels-Jørgen Jacobsen

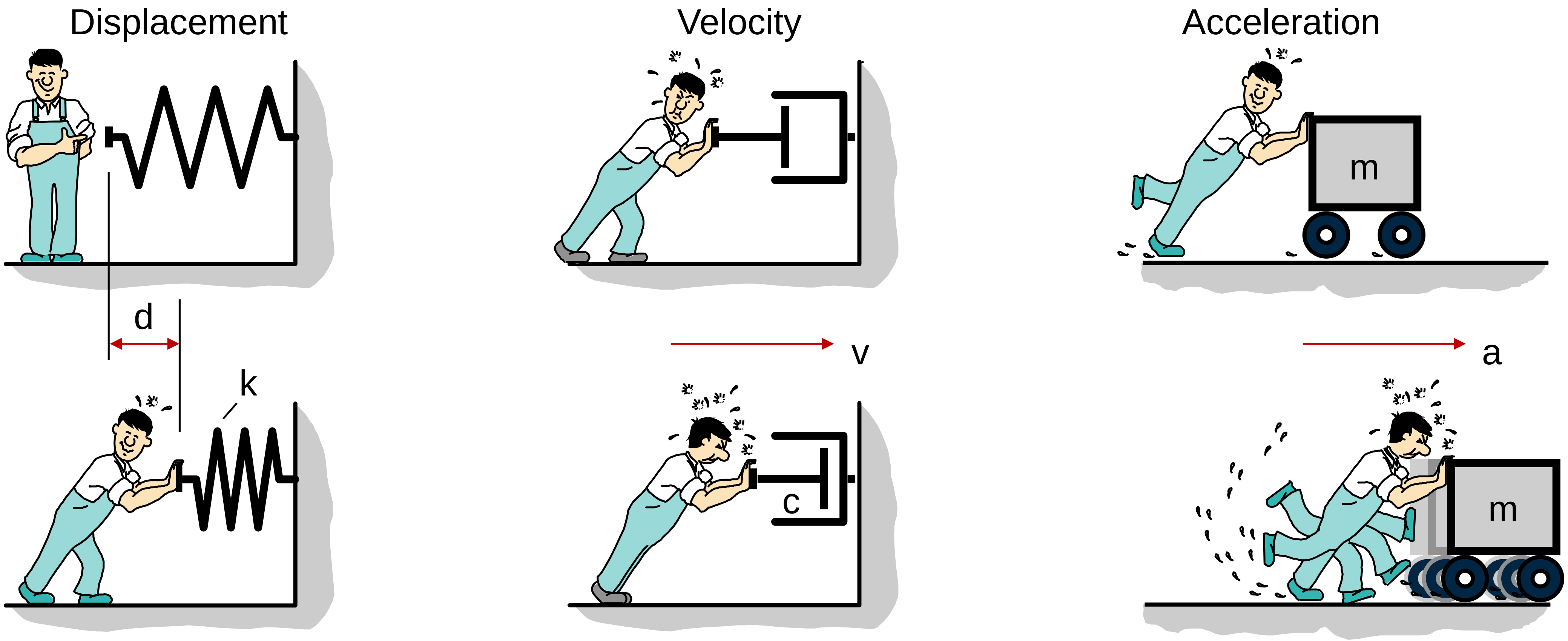
Product Manager – Structural Dynamics Solutions

February 19th, 2020

Agenda

1. **Mechanical systems:** Components and parameters
2. The **SDOF model** and the **equation of motion**
3. **What** structural dynamics is, **why** it is important to perform analysis of it, **when** it is normally done and **how** it is typically done
4. Difference between **testing** and **simulation** and how the combined use can be beneficial
5. Difference between **signal analysis** and **system analysis**
6. Overview of the **most frequently used applications**
7. **Important trends** in structural dynamics
8. **Additional information**
9. **Q&A**

Mechanical systems: Components and parameters



$F = k \times d$

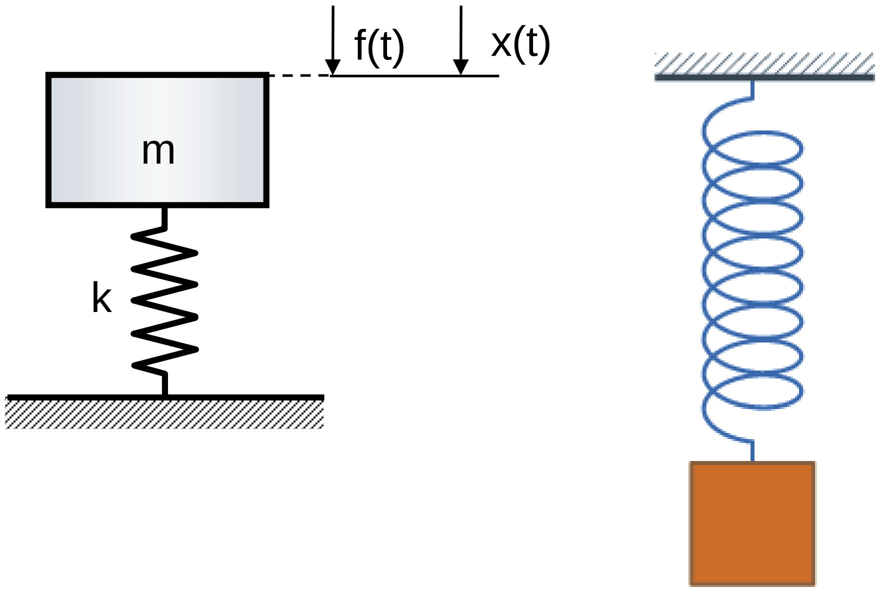
$F = c \times v$

$F = m \times a$

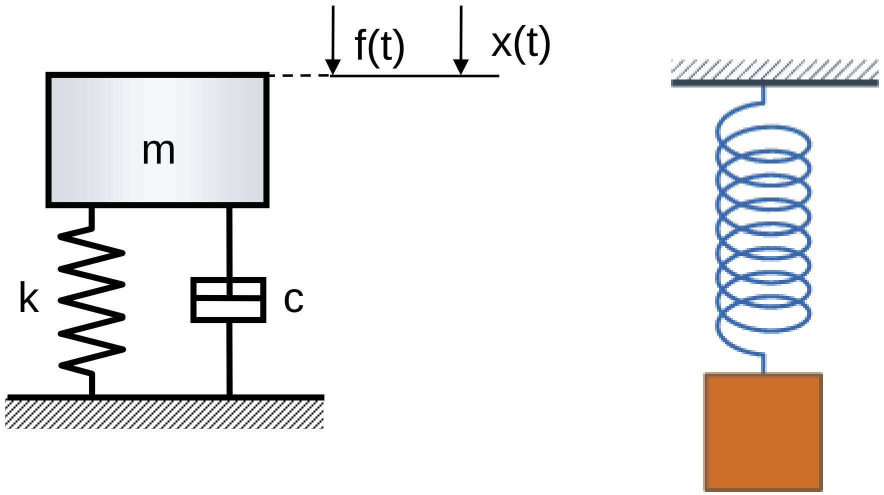
|

Single Degree of Freedom (SDOF) model

SDOF Model
Without Damping



SDOF Model
With Damping



Undamped natural (resonance) frequency:

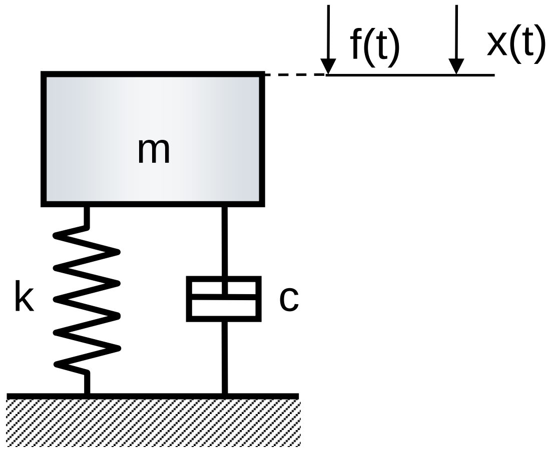
$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Damped natural (resonance) frequency:

$$f_d = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$$

Equation of motion – Time domain

Single Degree Of Freedom (SDOF) Model



$\ddot{x} = Acceleration$

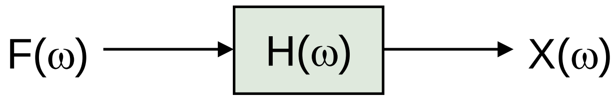
$\dot{x} = Velocity$

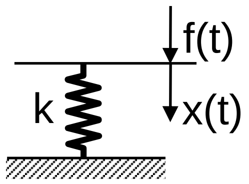
$x = Displacement$

$$f(t) = m\ddot{x}(t) + c\dot{x}(t) + kx(t)$$

Force Balance:
External = Inertial + Dissipative + Restoring

SDOF model – Transfer function

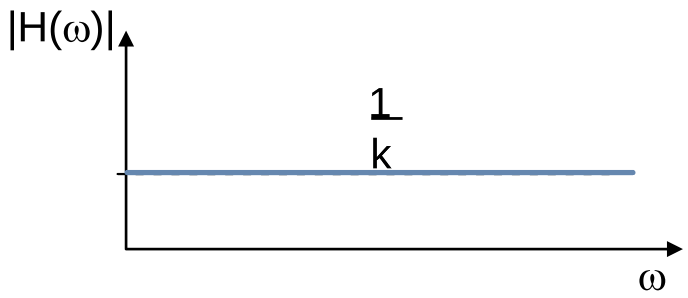


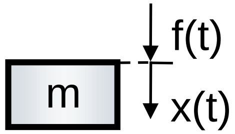


$$f(t) = k\,x(t)$$

$$F(\omega) = k\,X(\omega)$$

$$H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{k}$$

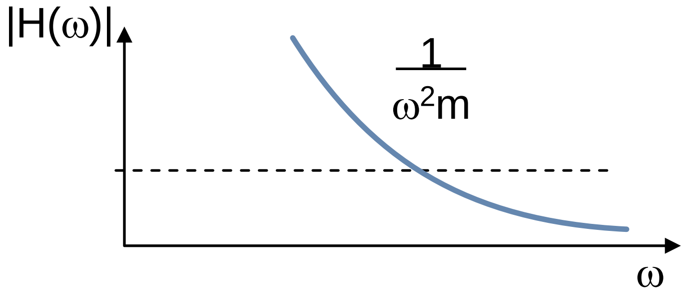


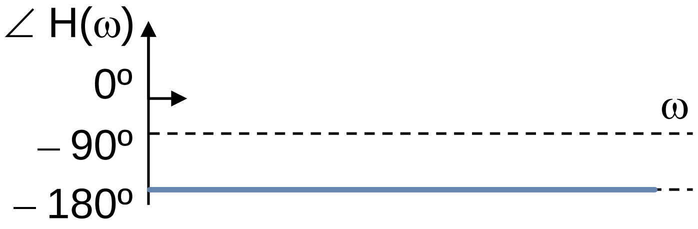


$$f(t) = m\ddot{x}(t)$$

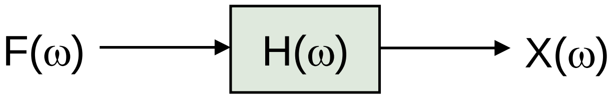
$$F(\omega) = -\,\omega^2\,mX(\omega)$$

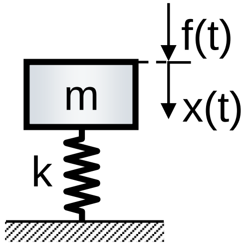
$$H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{-\,\omega^2 m}$$





SDOF model – Transfer function

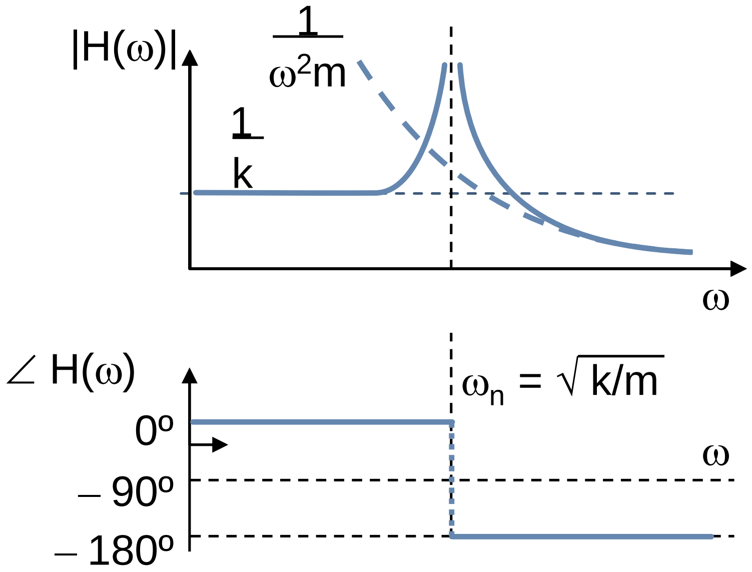


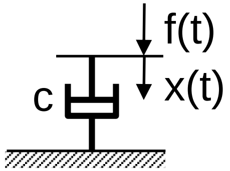


$$f(t) = m\ddot{x}(t) + kx(t)$$

$$F(\omega) = -\,\omega^2 mX(\omega) + kX(\omega)$$

$$H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{-\,\omega^2 m + k}$$

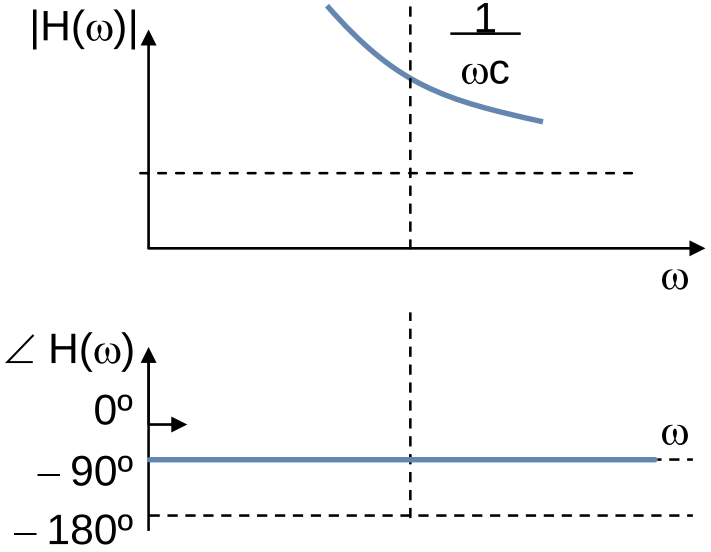




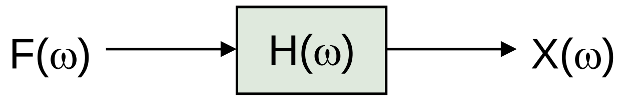
$$f(t) = c\dot{x}$$

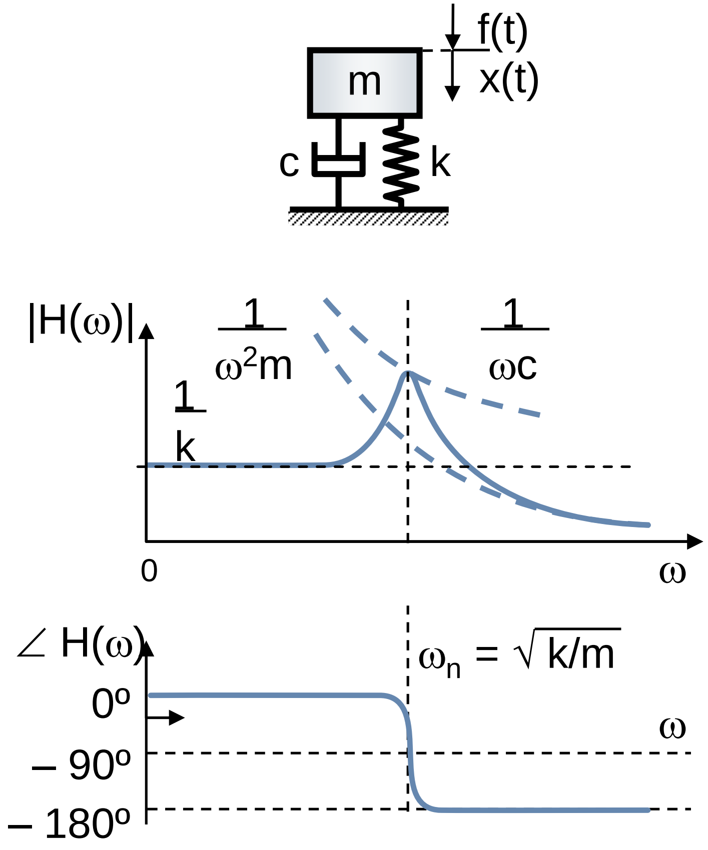
$$F(\omega) = j\omega cX(\omega)$$

$$\frac{X(\omega)}{F(\omega)} = \frac{1}{j\omega c}$$



SDOF model – Transfer function





Newton's 2nd law:

$$f(t) = m\ddot{x}(t) + c\dot{x}(t) + kx(t)$$

$$F(\omega) = -\omega^2 mX(\omega) + j\omega\, cX(\omega) + kX(\omega)$$

$$H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{-\omega^2 m + j\omega c + k}$$

Undamped natural frequency:

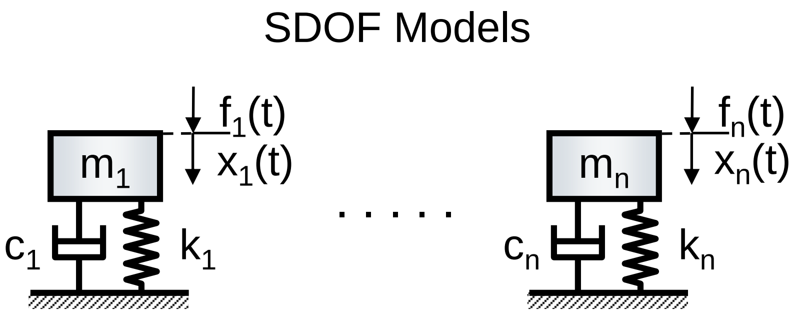
$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

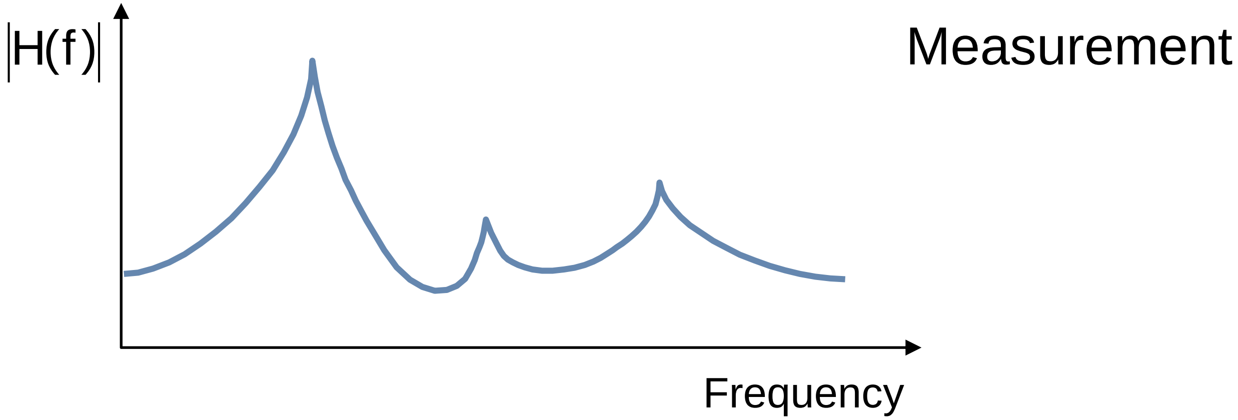
Damped natural frequency:

$$f_d = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$$

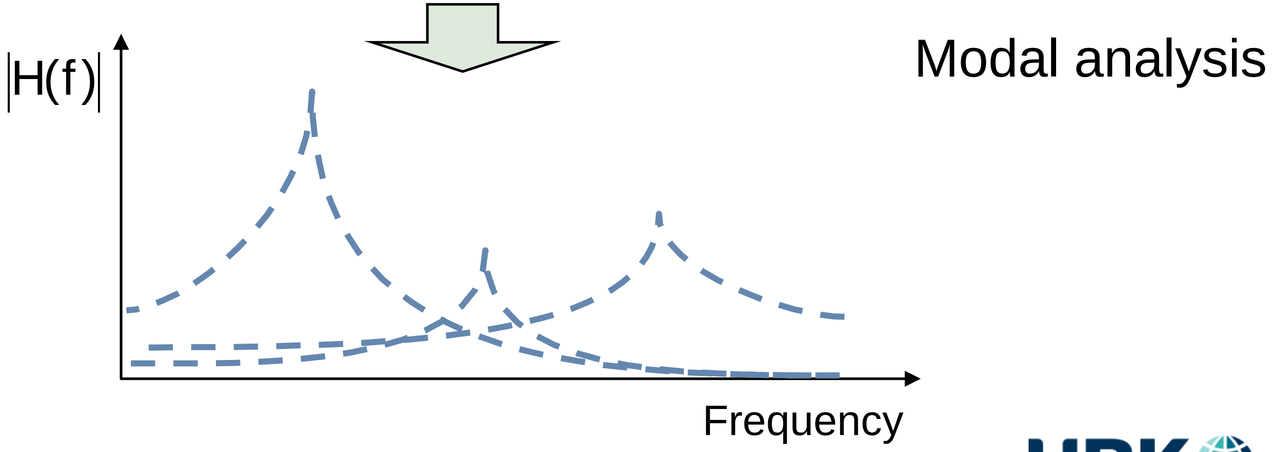
Multiple Degrees of Freedom (MDOF) model

MDOF model = Σ(SDOF models)





Curve-fitting
(pattern recognition)



Agenda

1. **Mechanical systems:** Components and parameters
2. The **SDOF model** and the **equation of motion**
3. **What** structural dynamics is, **why** it is important to perform analysis of it, **when** it is normally done and **how** it is typically done
4. Difference between **testing** and **simulation** and how the combined use can be beneficial
5. Difference between **signal analysis** and **system analysis**
6. Overview of the **most frequently used applications**
7. **Important trends** in structural dynamics
8. **Additional information**
9. **Q&A**

HBK HOTTINGER BRÜEL & KJÆR

What is structural dynamics measurements and analysis?

- **Observation** of the **actual dynamic behaviour** of a structure under certain **operating conditions**
  - **Deflection patterns** as a function of **time** or at specific **frequencies or orders (ODS analysis)**

- Creation of a mathematical model describing the **inherent dynamic properties** of a structure, so its **behaviour** can be **predicted** in various situations
  - The **modal parameters**: Natural frequency, damping ratio and mode shape **(Modal analysis)**

Mode shapes - Theory:

- A mode shape is real and all points and directions (DOFs) move either in-phase or out-of-phase
- The mode shapes are orthogonal, i.e. 100% independent

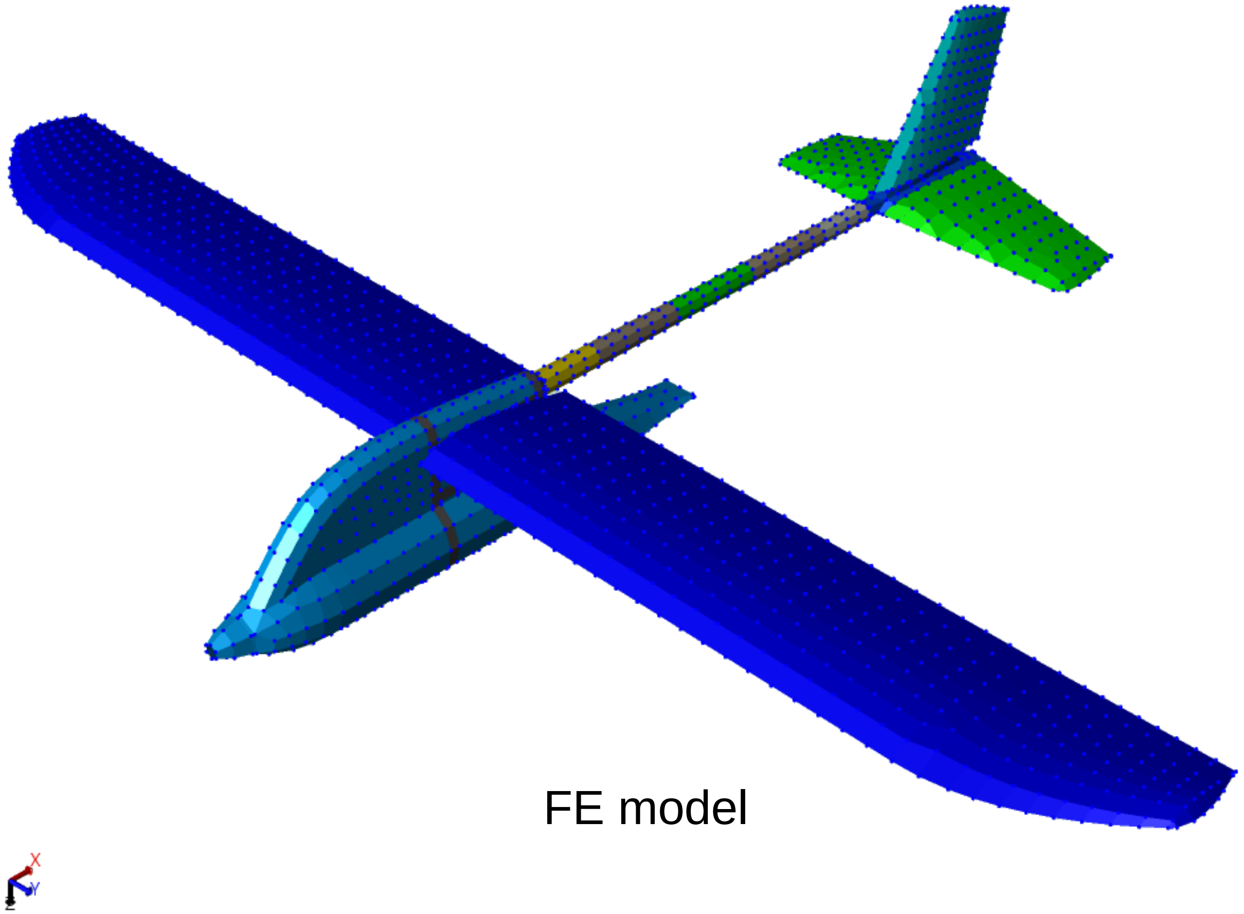
Mode Shapes - Practice:

- Mode shapes are complex due to, for example, non-optimal measurements and analysis and non-proportional damping in the structure

- Measurement and analysis techniques exists to obtain real mode shapes (Normal Mode Tuning)

Examples of mode shapes – Finite Element Analysis (FEA) of a small aircraft

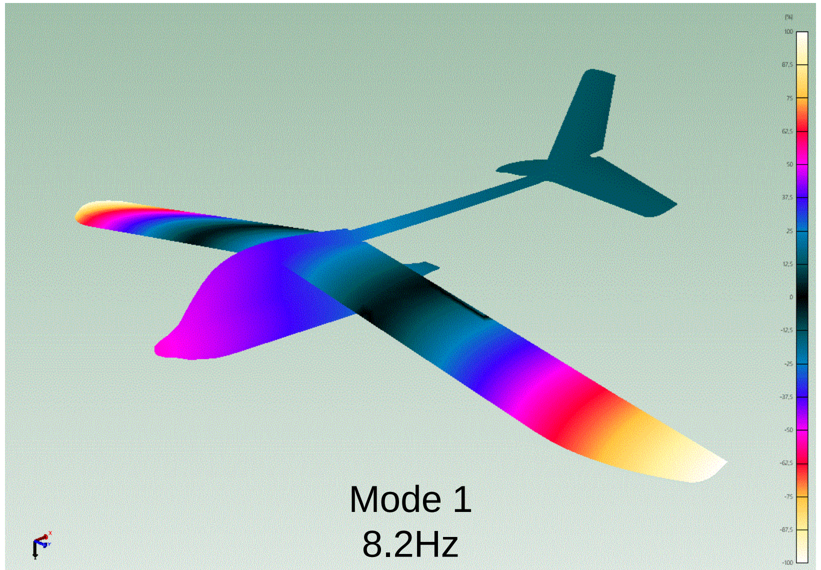
Wingspan: 3m
Length: 2m
Weight: 17kg

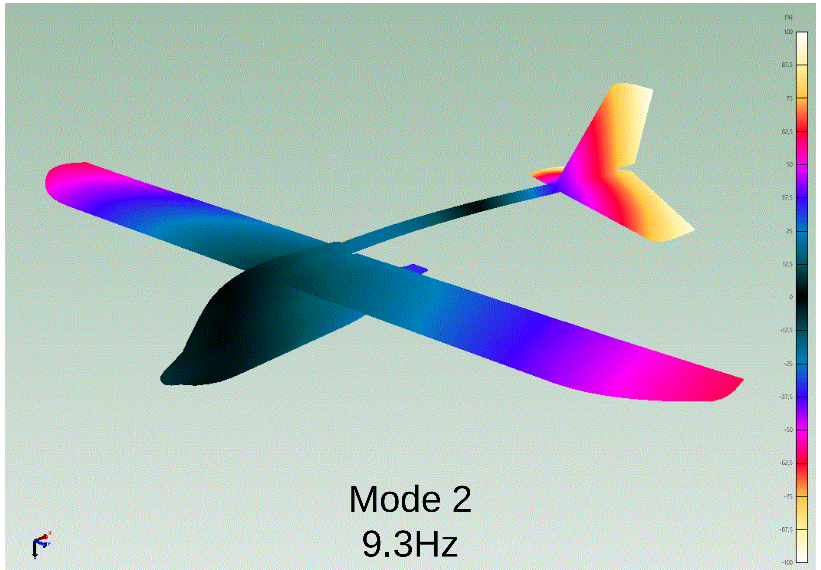


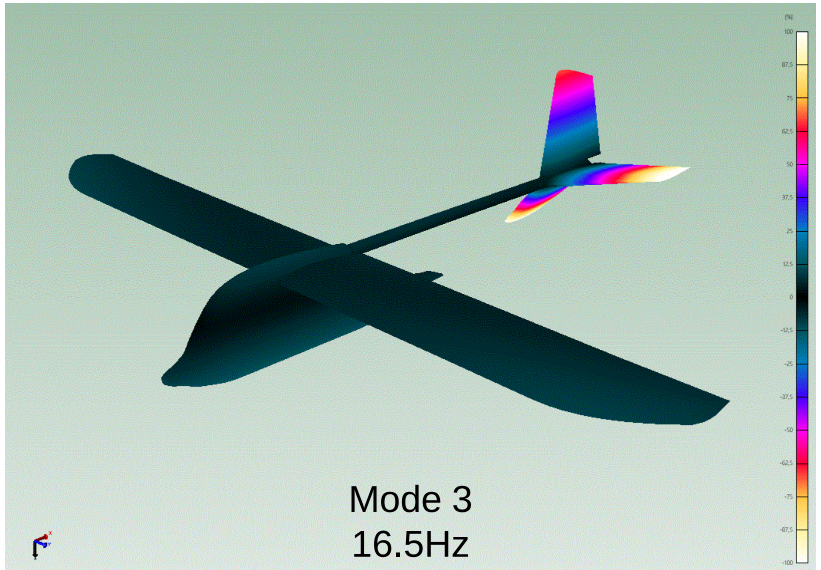
FE model

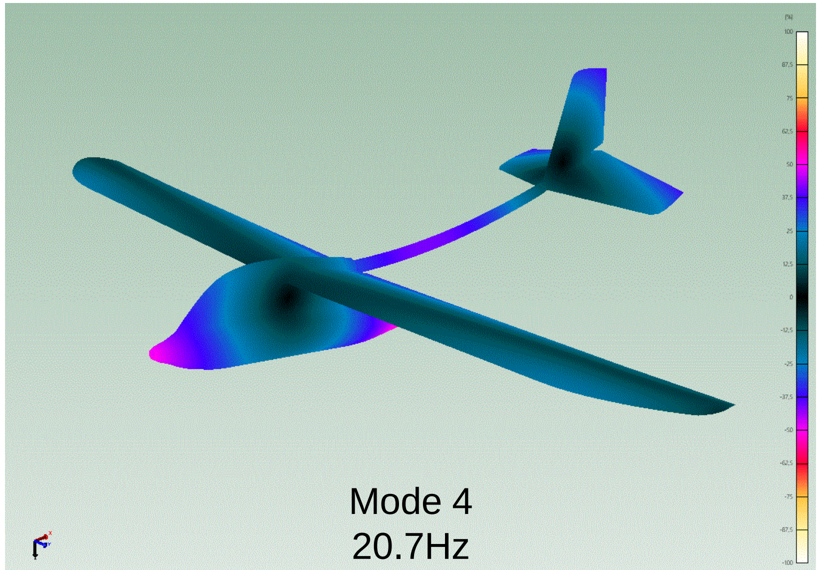
HBK HOTTINGER BRÜEL & KJÆR

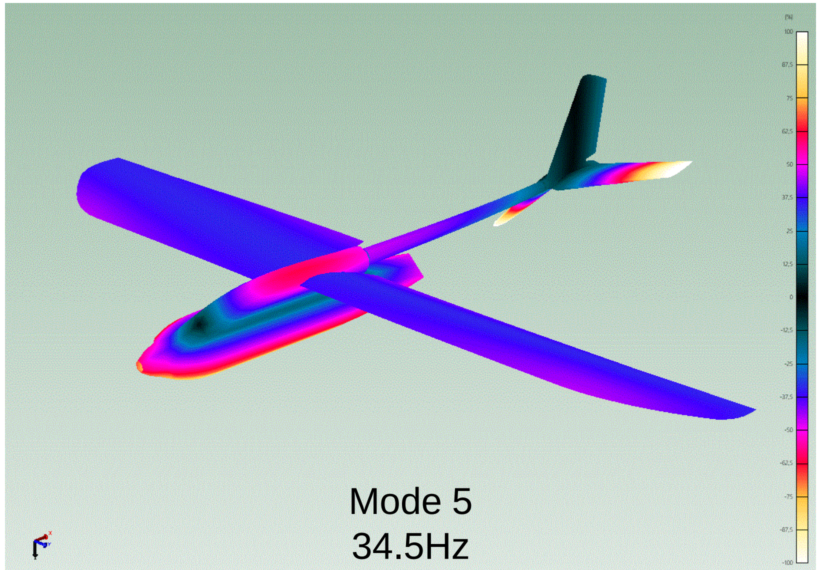
Examples of mode shapes – Finite Element Analysis (FEA) of a small aircraft

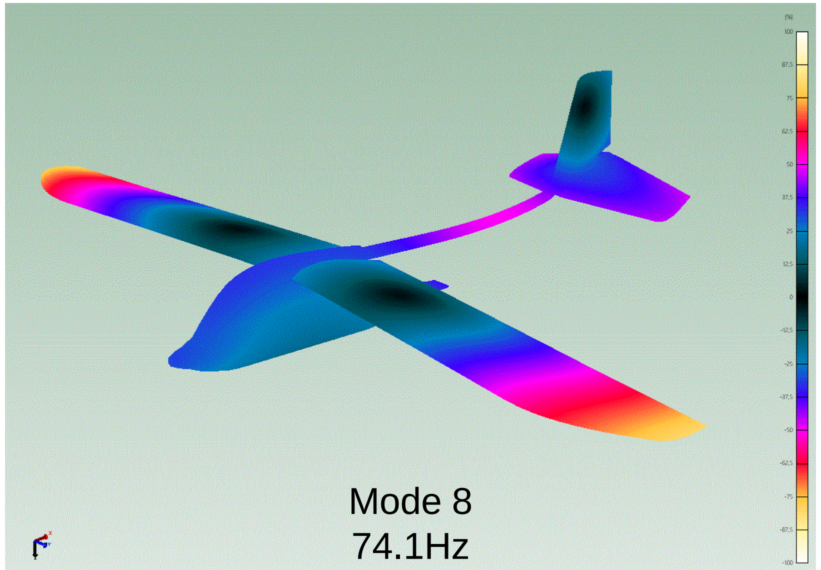




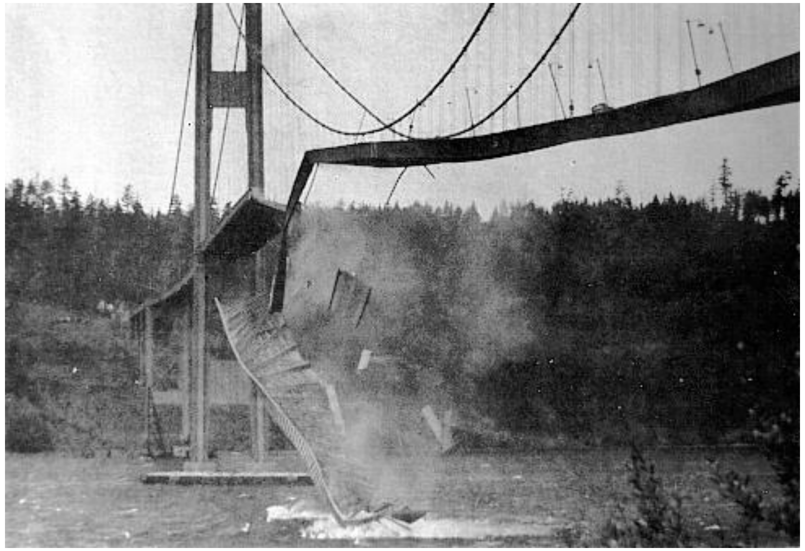








Flutter - Tacoma Narrows Bridge, Nov. 7th, 1940



Aircraft Flutter

Why is structural dynamics so important?

To understand in order to **improve** safety, reliability, performance, consumption and comfort

Structural dynamics is a key discipline within sound & vibration

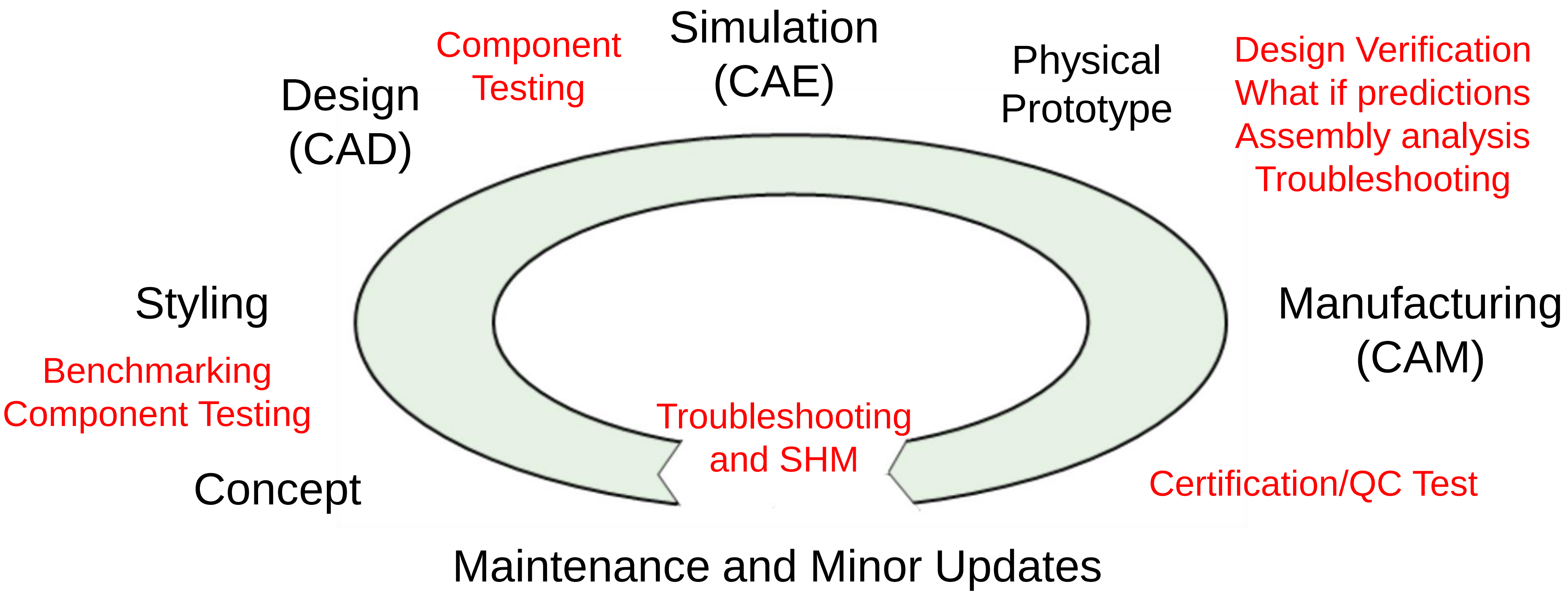
- Aerospace & Defence
- Automotive & Trains
- Shipbuilding
- Power Generation
- Civil Engineering
- Heavy Industry / Rotating Machinery
- Consumer Products
- Research & Education
- And many more …

HBK
HOTTINGER BRÜEL & KJÆR

When do we perform analysis of structural dynamics?

- Design verification, certification testing, QC testing, troubleshooting, prediction of “what if” scenarios, assembly analysis, benchmarking, structural health monitoring and much more

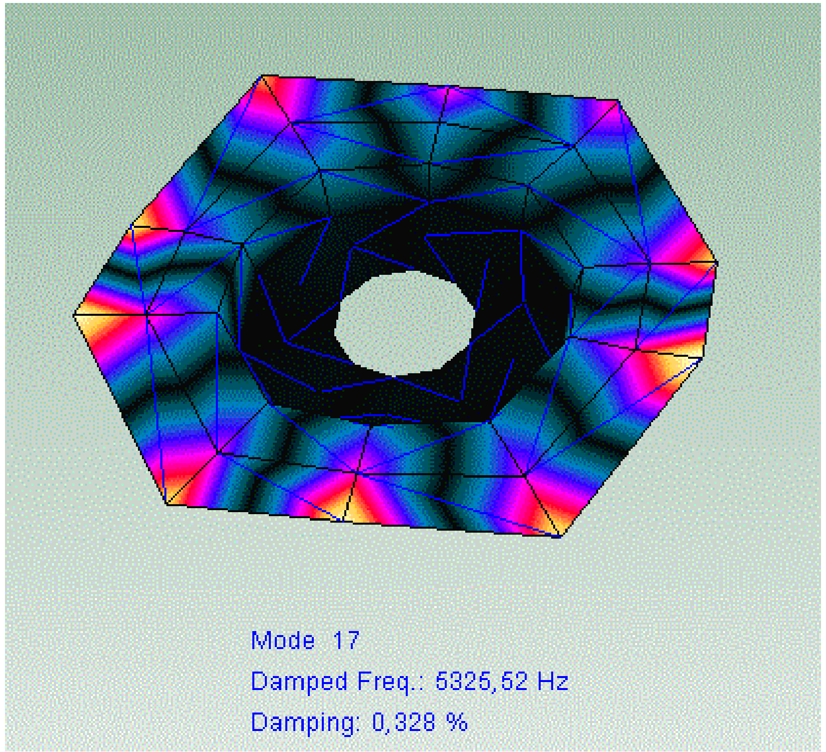
Product Development Cycle

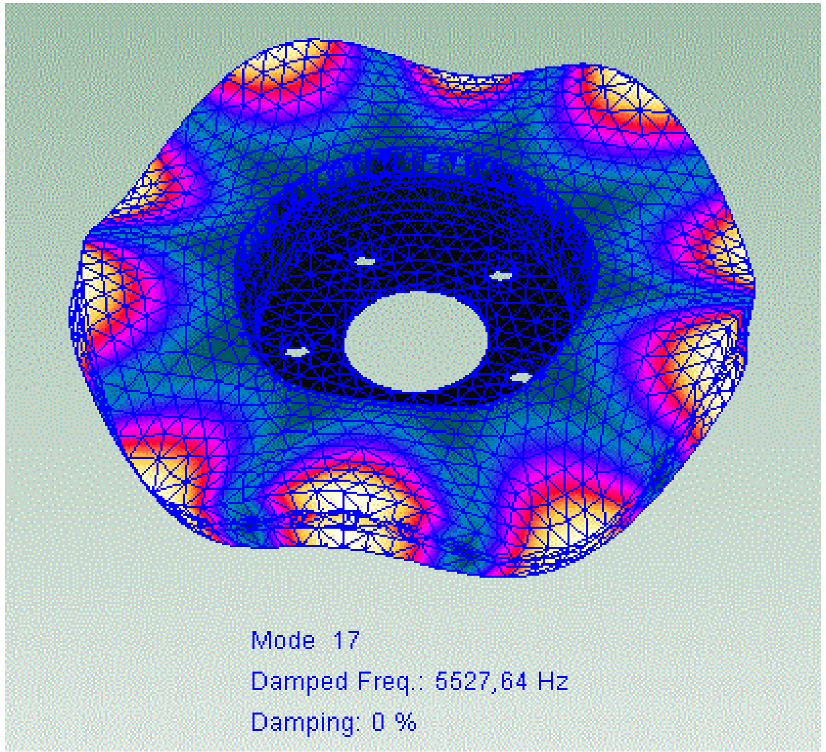


CAD: Computer Aided Design
CAE: Computer Aided Engineering
CAM: Computer Aided Manufacturing
QC: Quality Control
SHM: Structural Health Monitoring

How do we perform analysis of structural dynamics?

- Based on **measurements** also known as **experimental testing** or based on computer **simulation** models also known as **analytical simulation**
- The simulation results are typically correlated with the test results using **model correlation** to update and refine the simulation models to provide better predictions



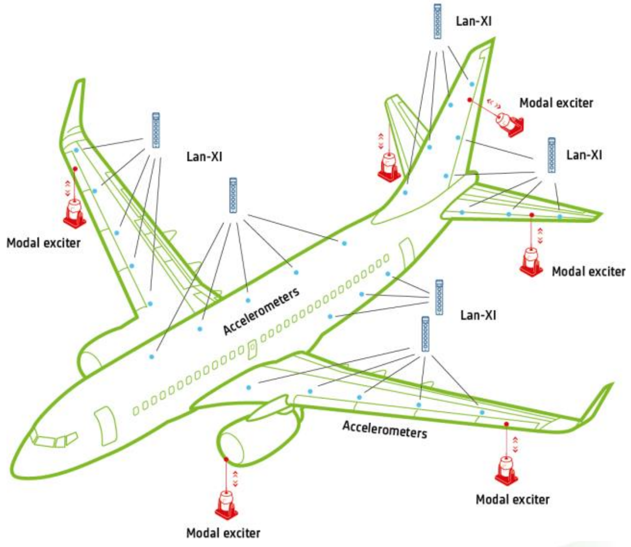


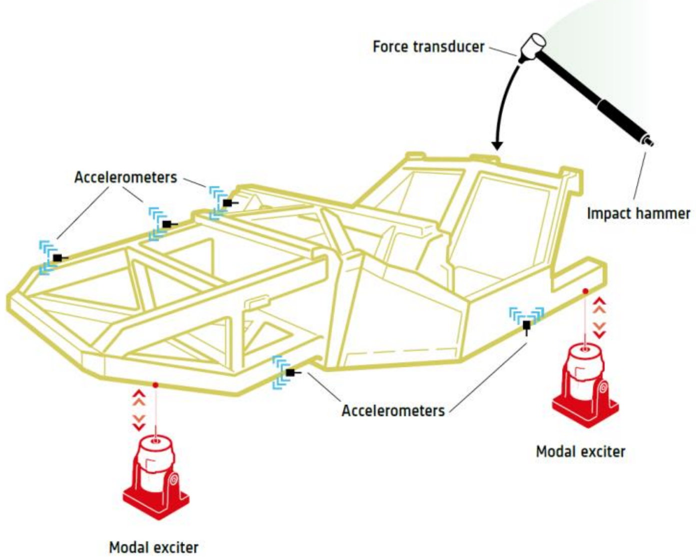
Results animation of a brake disc (mode 17)

- Left: Testing based on reduced test model
- Right: Simulation based on large Finite Element Model (FEM)

How to perform structural dynamics measurements

- The structure is excited by **unmeasured natural forces** (ODS[1], OMA[2]) or **measured applied forces** (classical modal analysis) in one or more points and directions (Degrees-of-Freedoms: **DOFs**)
- Forces are either applied using a dedicated **impact hammer** that also measures the applied force, or by using one or more **modal exciters (shakers)** and measuring the applied forces using force transducers
- The result is measured in a number of response DOFs, typically using uniaxial or triaxial **accelerometers**
- Tests are performed from **2-channel impact testing** using, for example, one hammer and one accelerometer, to large modal surveys with more than **10 modal exciters** and **1000 response DOFs**





1) Operating Deflection Shapes
2) Operational Modal Analysis

Agenda

1. **Mechanical systems:** Components and parameters
2. The **SDOF model** and the **equation of motion**
3. **What** structural dynamics is, **why** it is important to perform analysis of it, **when** it is normally done and **how** it is typically done
4. Difference between **testing** and **simulation** and how the combined use can be beneficial
5. Difference between **signal analysis** and **system analysis**
6. Overview of the **most frequently used applications**
7. **Important trends** in structural dynamics
8. **Additional information**
9. **Q&A**

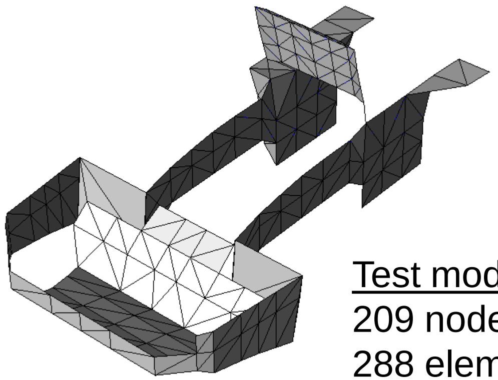
HBK HOTTINGER BRÜEL & KJÆR

Testing

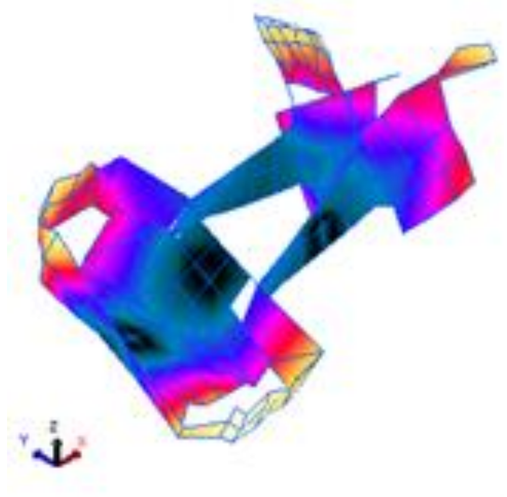
Characterization of the *structural properties* and/or *behaviour*
of structures obtained by *experimental means (testing)*

Methods

- In-operation measurements (in situ) during (approximated) real-life excitation and actual boundary conditions (for example, ODS and OMA)
- Controlled measurements (laboratory) with controlled excitation and controlled boundary conditions (for example, Classical Modal Analysis)



Test model
209 nodes
288 elements



Advantages

- Characterization of actual physical structure – Good confidence in results
- Easy and fast to perform measurements and do post-analysis
- Relative inexpensive instrumentation – in particular for smaller setups

Disadvantages

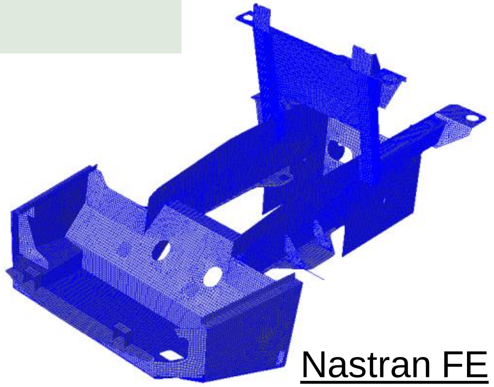
- Requires available physical test objects
- Becomes time-consuming and expensive if many modifications are required

Simulation (Finite Element Analysis) – *The Digital Twin*

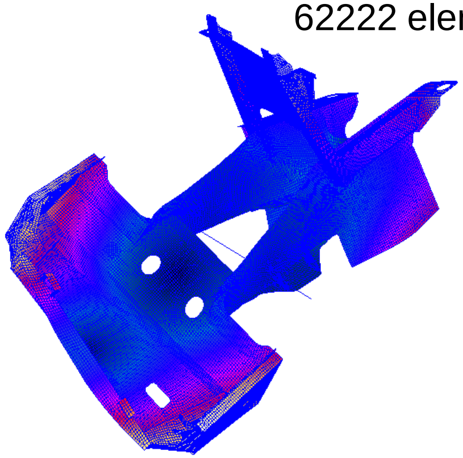
Characterization of the *structural properties* and/or *behaviour* of structures using *simulation models*

Methods

- Analysis of numerical models based on disjoint components with material properties called Finite Elements (FEs) using Finite Element Analysis (FEA) programs (for example, Nastran®, Ansys®, Abaqus® and I-deas)



Nastran FE model
62276 nodes
62222 elements



Advantages

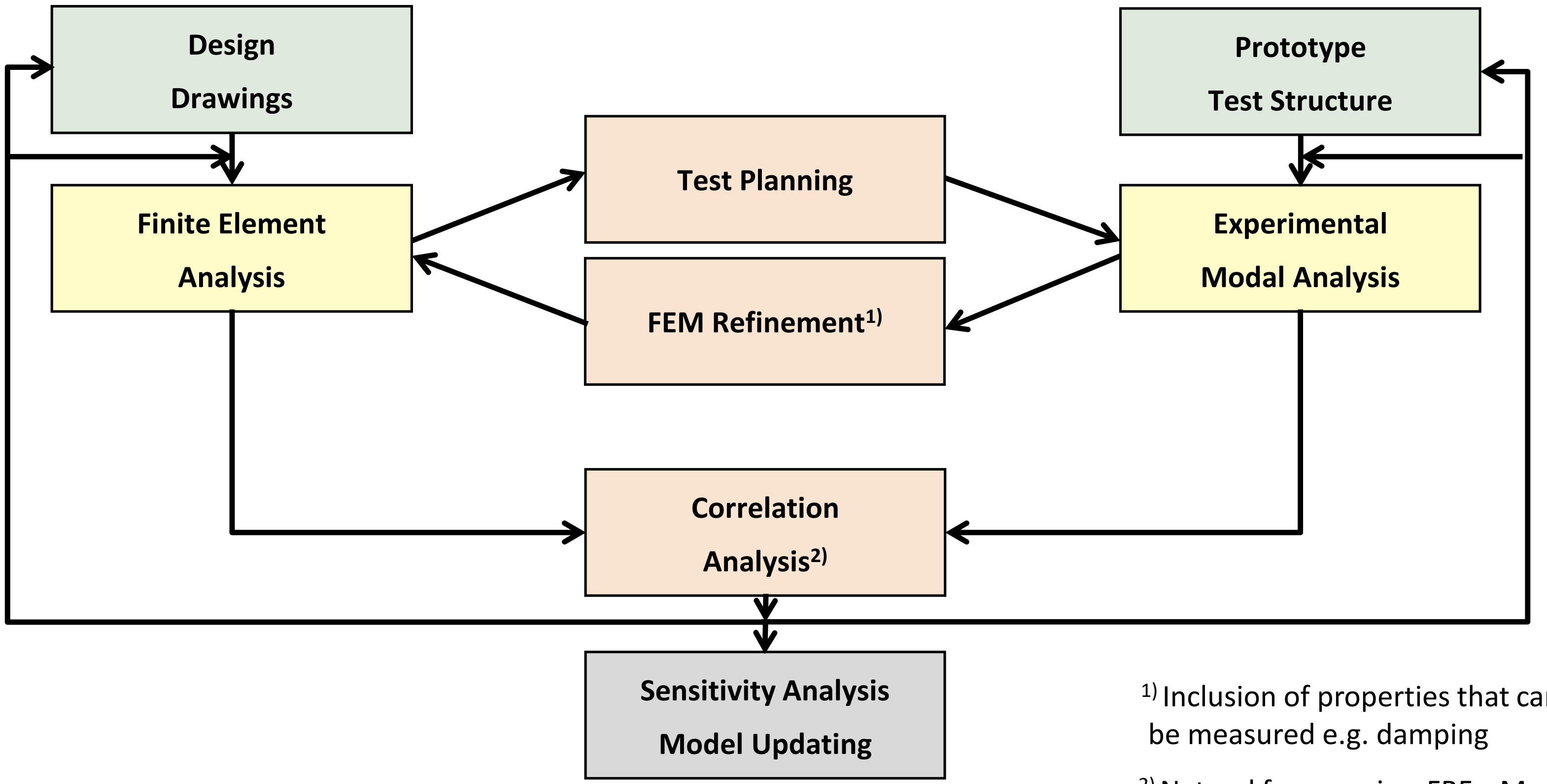
- Shortens time from concept to production
- Reduces the number of required prototypes in product development

Disadvantages

- Expensive systems requiring high degree of expert knowledge
- Does often not include (accurate) damping information
- Accuracy of initial models often insufficient
- Simulations can be quite time-consuming

Test-FEA Integration – Schematic Overview

Shorten time from concept to production by *optimizing strategies* for *testing models* and *improving* the development of *FE models*



1) Inclusion of properties that can only be measured e.g. damping

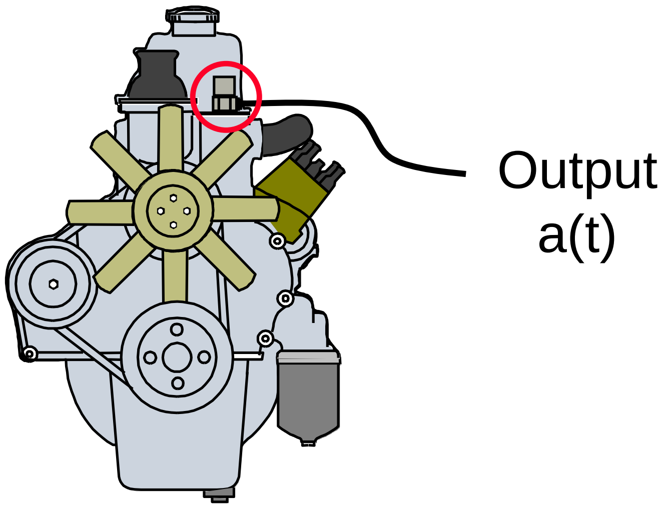
2) Natural frequencies, FRFs, Mode shapes (MAC and Orthogonality)

Agenda

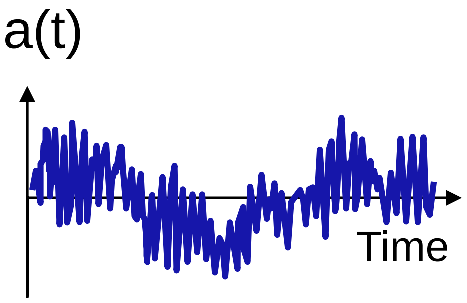
1. **Mechanical systems:** Components and parameters
2. The **SDOF model** and the **equation of motion**
3. **What** structural dynamics is, **why** it is important to perform analysis of it, **when** it is normally done and **how** it is typically done
4. Difference between **testing** and **simulation** and how the combined use can be beneficial
5. Difference between **signal analysis** and **system analysis**
6. Overview of the **most frequently used applications**
7. **Important trends** in structural dynamics
8. **Additional information**
9. **Q&A**

HBK HOTTINGER BRÜEL & KJÆR

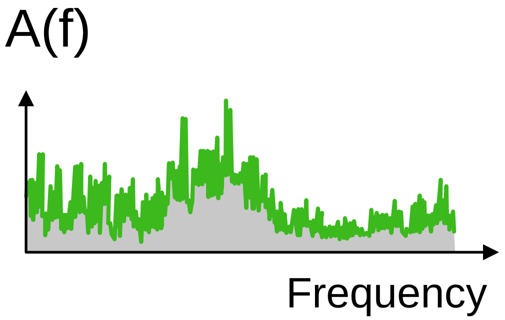
Signal Analysis - Definition



Time signal



Frequency spectrum



No model created – Only observation of the output !

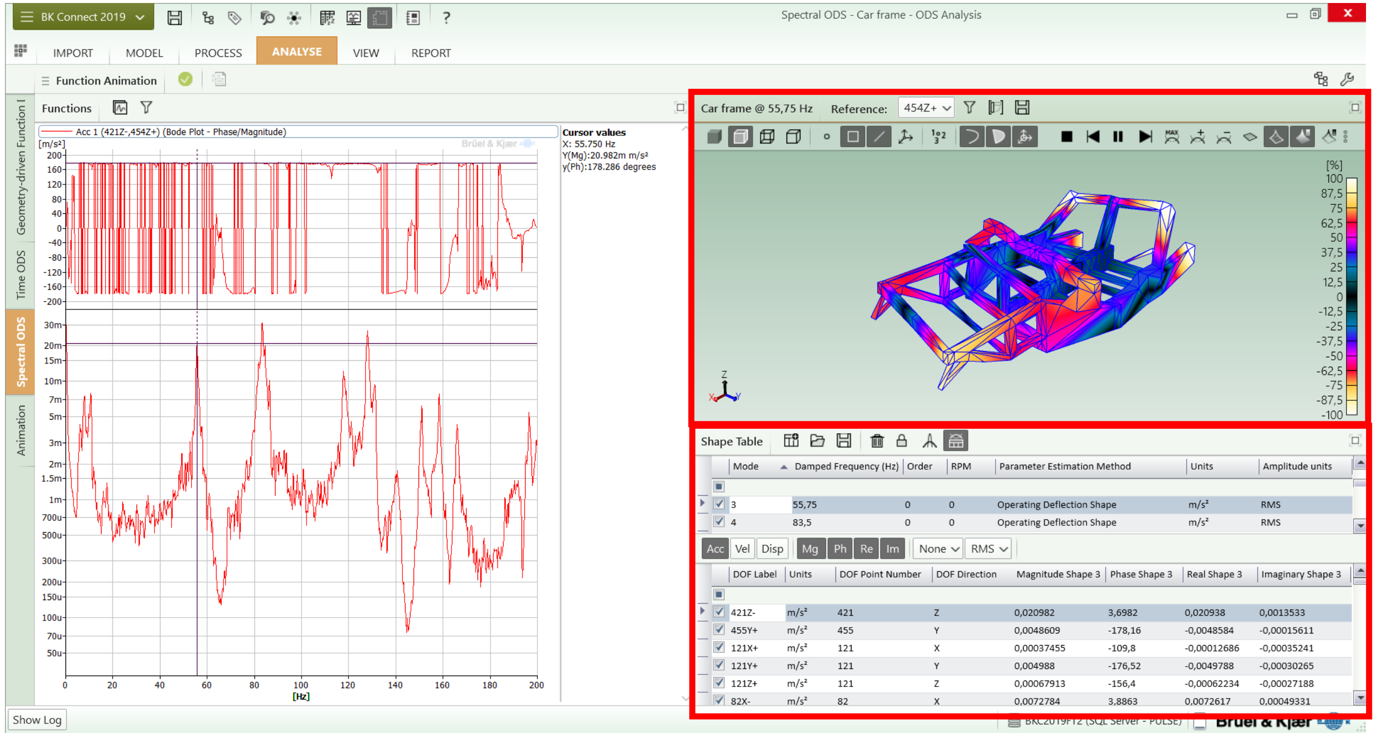
Example of Signal Analysis – ODS Analysis

Determination and visualization of the vibration pattern of structures under operating conditions

Operating conditions
- Speed, Load, Temp., Pressure, Flow, ...

Vibration signals
- Stationary
- Quasi-stationary
  - » Slightly varying speed or excitation level
  - » Run-up/down tests
- Transient

Results
- Geometry animated in different DOFs
- Table of Shapes for different frequencies/orders
  - Acceleration, Velocity and Displacement for each shape
- Table of deflection pattern at different time instances



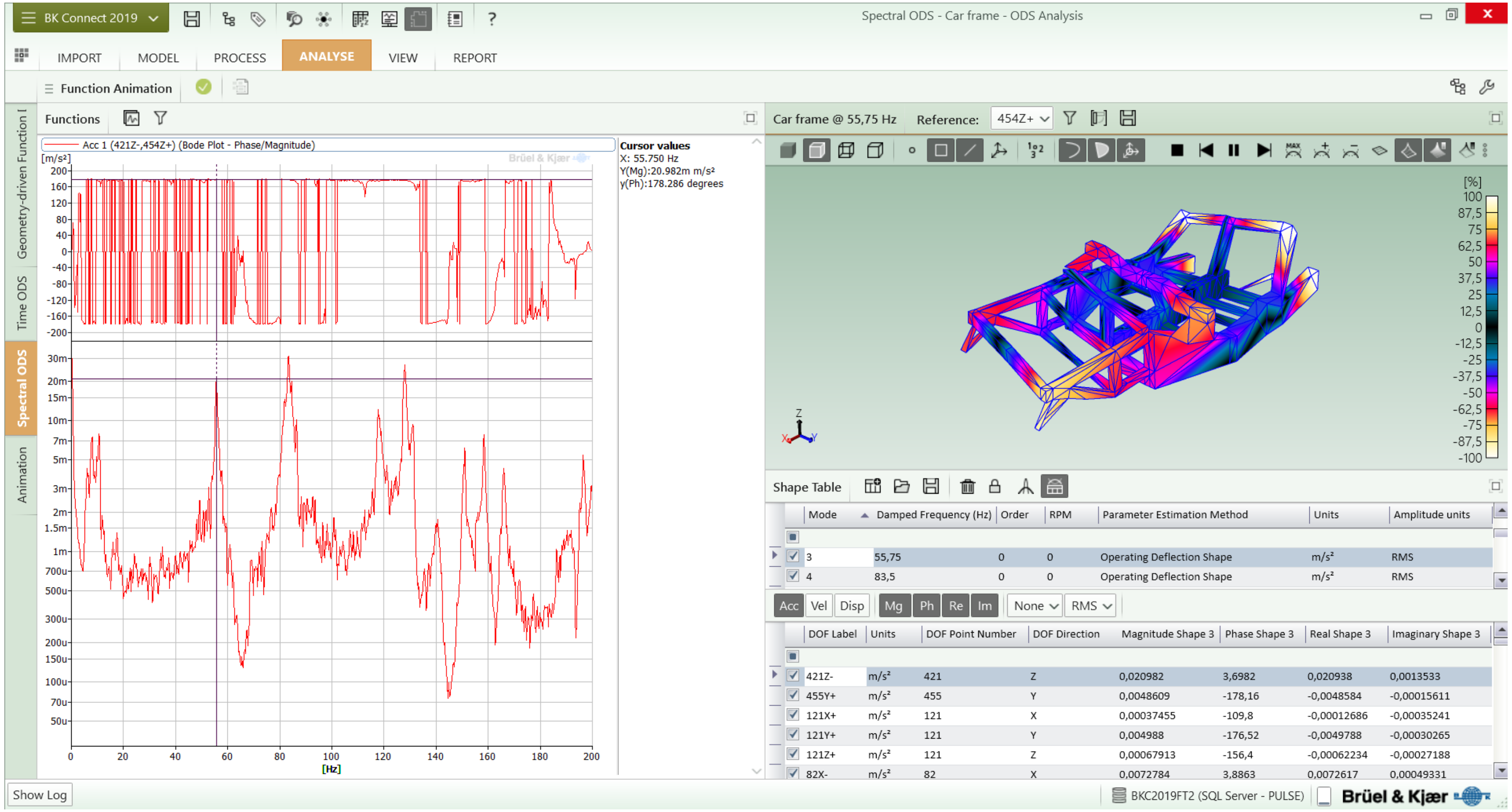
ODS analysis answers the questions:

How does the structure actually vibrate?

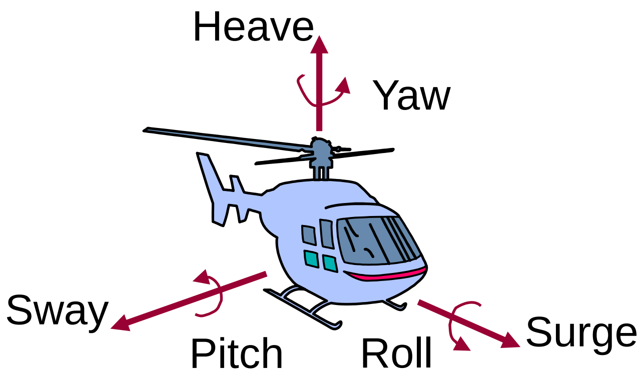
What is the actual absolute motion of one DOF vs. another?

Brief Demo – Spectral ODS (Frequency ODS) of a Car Frame

- Sports car frame excited by finger tapping etc.
- Measurement made using 20 triaxial accelerometers
- Triax reference in left front corner
- Interpolation in 475 nodes
- Frequency range: 200Hz

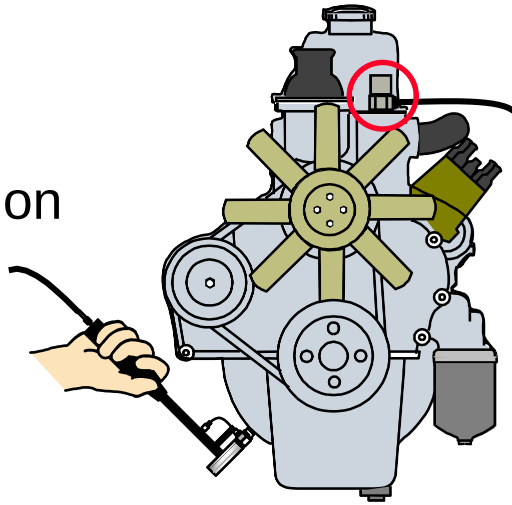


Rigid Body Modes (RBM)

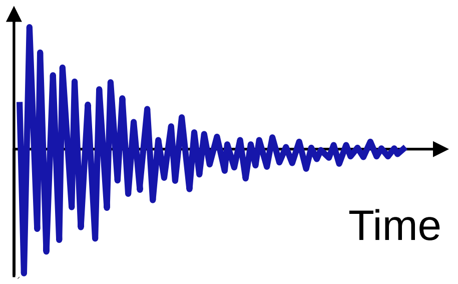


HBK HOTTINGER BRÜEL & KJÆR

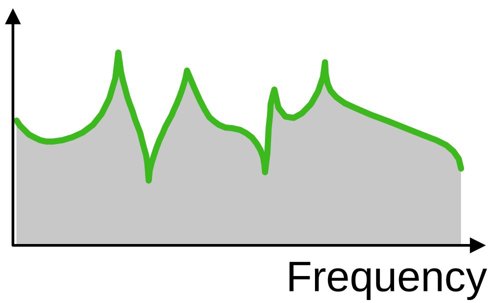
System Analysis - Definition



Impulse Response Function



Frequency Response Function



A model is created – Determination of the system's inherent properties !

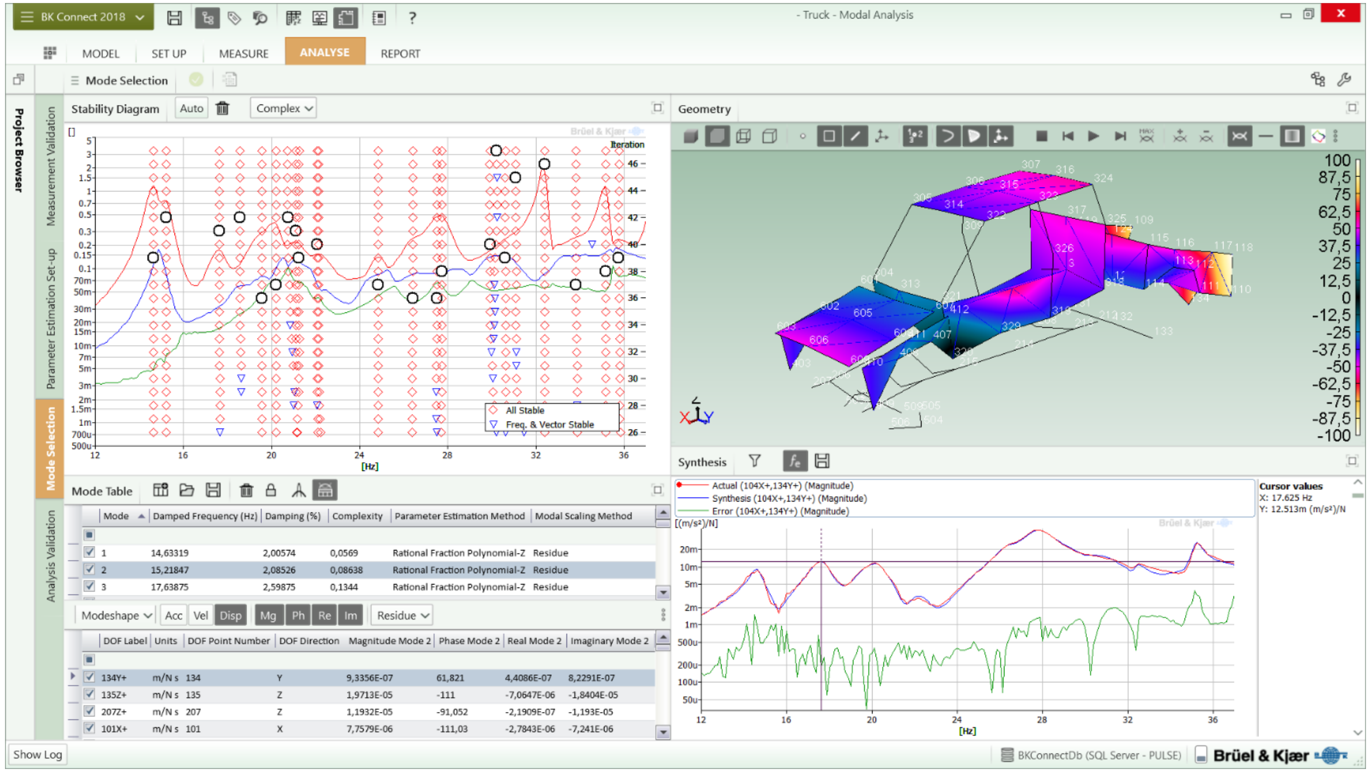
Example of System Analysis – Classical Modal Analysis (Modal Testing)

To obtain a mathematical model of the dynamic properties of a structure by experimental means using hammer or shaker excitation *(modal testing)*

Controlled measurements
- Boundary conditions
- Excitation signals
- Environmental conditions

Excitation signals
- Sinusoidal (fixed, sweep, step)
- Random (normal, burst, pseudo …)
- Transient (impact)
- …

Results – Modal Parameters
- Natural Frequencies
- Modal Damping
- Mode Shapes



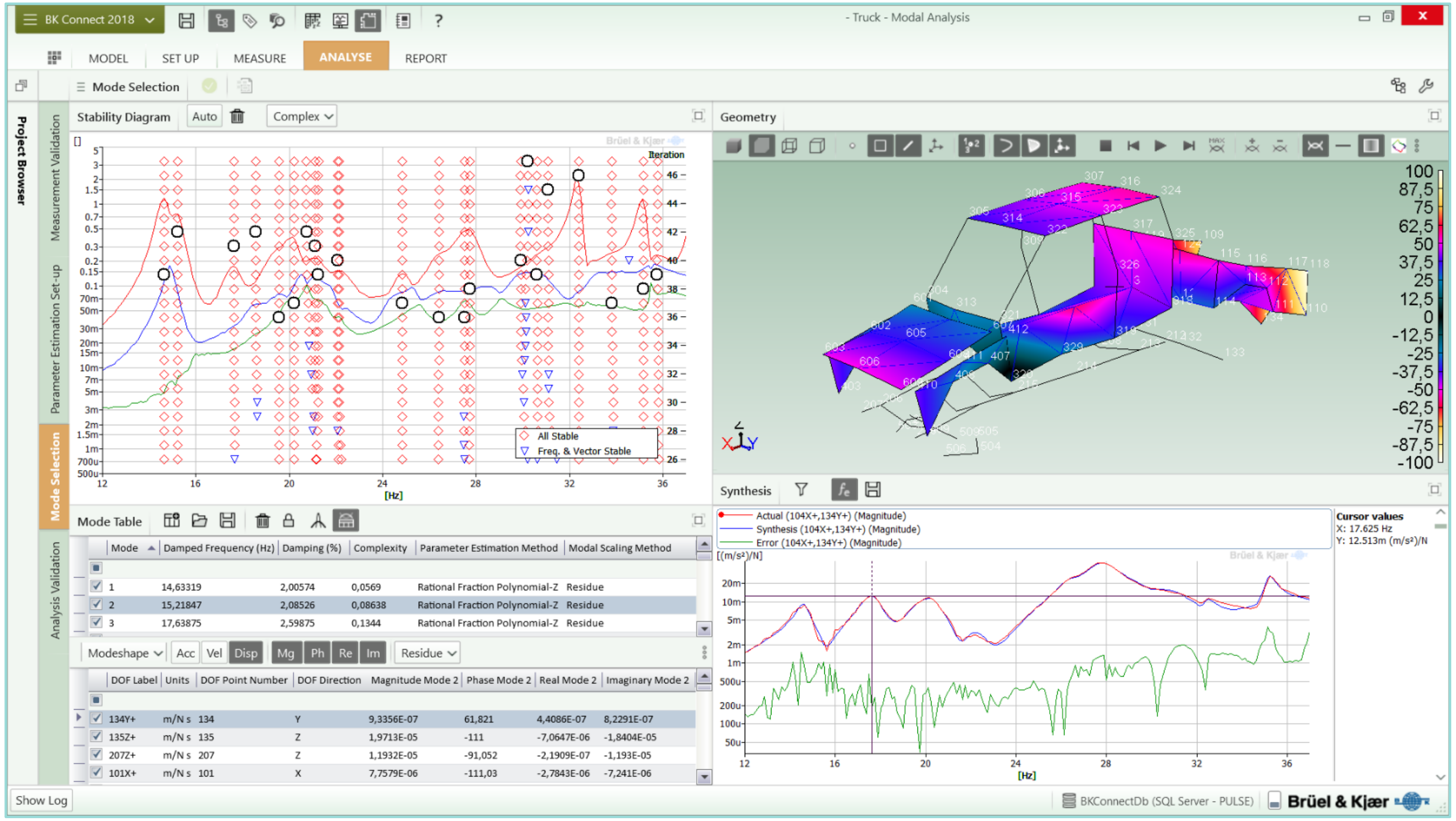
Classical Modal Analysis answers the question:

What is the structure’s inherent dynamic properties?

Brief Demo – Classical Modal Analysis of a Partly Trimmed Truck

Configuration

- Partly trimmed truck (heavily damped modes, non-linear behaviour)
- 3 shakers and 330 response DOFs (110 triax (4524-B)) => 990 FRFs



HBK
HOTTINGER BRÜEL & KJÆR

Agenda

1. **Mechanical systems:** Components and parameters
2. The **SDOF model** and the **equation of motion**
3. **What** structural dynamics is, **why** it is important to perform analysis of it, **when** it is normally done and **how** it is typically done
4. Difference between **testing** and **simulation** and how the combined use can be beneficial
5. Difference between **signal analysis** and **system analysis**
6. Overview of the **most frequently used applications**
7. **Important trends** in structural dynamics
8. **Additional information**
9. **Q&A**

HBK HOTTINGER BRÜEL & KJÆR

Key applications in Structural Dynamics

1) Finite Element (simulation)
2) Finite Element Analysis (simulation)
3) Frequency Response Functions

Test Planning

- Decimation of large FE[1] geometry models down to smaller test geometry models
- Selection of the optimum number and locations for excitation and response DOFs based on FEA[2] results

Operating Deflection Shapes analysis

- Determination of the vibration patterns of a structure under given operating conditions. Results are shown in tables and as animated geometries

Linearity studies and control

- Determination of the degree and type of non-linearity and how to control/handle it during (modal) measurements. Performed using swept or stepped sine testing

Classical Modal Analysis

- Determination and validation of a structure’s modal parameters (natural frequencies, damping estimates and mode shapes) from measured FRFs[3] using hammer or shaker testing

Structural Dynamics Modification

- Prediction of what the modal parameters will be of a modified structure before physically modifying it

Key applications in Structural Dynamics

Normal Mode Tuning

- Modal parameters found from multiple shaker testing by forcing the structure to respond in its normal (real) modes – one at a time. Used in Ground Vibration Testing (GVT) of aircraft for flutter predictions

Operational Modal Analysis

- Modal parameter extraction and validation under operating conditions – and often in-situ – by only measuring the responses of the structure

Structural Health Monitoring

- Damage detection, assessment and localization. To optimize maintenance services and improve expected lifetime predictions of civil engineering and mechanical structures

Model Correlation

- A visual and numerical correlation analysis of two modal models. Typically test versus FEA, but can also be test versus test, or FEA versus FEA

Model Updating

- Updating of FE models based on test results, so the FE models provide better predictions

Agenda

1. **Mechanical systems:** Components and parameters
2. The **SDOF model** and the **equation of motion**
3. **What** structural dynamics is, **why** it is important to perform analysis of it, **when** it is normally done and **how** it is typically done
4. Difference between **testing** and **simulation** and how the combined use can be beneficial
5. Difference between **signal analysis** and **system analysis**
6. Overview of the **most frequently used applications**
7. **Important trends** in structural dynamics
8. **Additional information**
9. **Q&A**

Trends in Structural Dynamics

- Much closer link between test and simulation. Design verification is becoming one of the main reasons for structural testing in an increasing number of market segments.
- More accurate, larger and complex tests are required to update continuously improved FE models. Total test time is increasing.
- Testing of simple structures is becoming a low-level task performed by technicians. Increased requirements for ease-of-use, automation, efficiency and reliability.
- Development processes are becoming shorter, more integrated, more automated and performed in parallel. Consequently, testing is performed at more stages - and levels - during the development process.
- Statistical uncertainty analysis to assess the quality and robustness of tests.
- Increasing focus on in-operation structural measurements (ODS and OMA) to perform/simulate “real-life” conditions and to perform tests previously impossible to do.
- Increasing focus on Structural Health Monitoring (SHM) for damage detection, condition-based maintenance and lifetime prediction. From being mainly a civil engineering discipline it is now increasingly used in aerospace, space, wind energy and for large machinery. Strong interest in automotive and other markets.
- Increasing focus and research in non-linear system analysis due to increasing use of, for example, composite and active materials. Techniques have significantly improved in recent years, but are, for general purposes, still not robust enough.

Agenda

1. **Mechanical systems:** Components and parameters
2. The **SDOF model** and the **equation of motion**
3. **What** structural dynamics is, **why** it is important to perform analysis of it, **when** it is normally done and **how** it is typically done
4. Difference between **testing** and **simulation** and how the combined use can be beneficial
5. Difference between **signal analysis** and **system analysis**
6. Overview of the **most frequently used applications**
7. **Important trends** in structural dynamics
8. **Additional information**
9. **Q&A**

HBK HOTTINGER BRÜEL & KJÆR

Previous and upcoming 1h webinars on structural dynamics

<u>Nov. 20, 2019</u>

- Operational Modal Analysis – Modal Parameter Identification under Operating Conditions
  - Recording available on: https://www.hbm.com/en/8507/webinar-operational-modal-analysis-modal-parameter-identification-under-operating-conditions/

<u>Feb. 20, 2020</u>

- Operating Deflection Shapes Analysis – Determination of Vibration Patterns under Operating Conditions
  - Sign up on: https://www.hbm.com/en/8740/ods-vibration-patterns-under-operating-conditions/

<u>May 15, 2020</u>

- Classical Modal Analysis using Hammer and Shaker Excitation
  - To be published soon!

Further Information - Brüel & Kjær

- Knowledge Centre (www.bksv.com/Knowledge-center)
  - Case Studies, Application Notes, Technical Reviews, Conference Papers, Primers and Handbooks
- Training (www.bksv.com/Training)
  - Courses, Webinars, Customized Training, Video Tutorials
    - Includes 3-day course on Classical Modal Analysis and 2-day course on Operational Modal Analysis
- Structural Dynamics - Product and Application pages
  - Operational Modal Analysis, Classical Modal Analysis, ODS Analysis, Structural Health Monitoring, Test-FEA Integration …
  - Software, hardware, transducers, modal exciters, impact hammers …
  - www.bksv.com/Applications/product-vibration/structural-dynamics
  - www.bksv.com/products/Analysis-software/structural-dynamics-software
  - www.bksv.com/products/data-acquisition-systems-and-hardware/LAN-XI-data-acquisition-hardware
  - www.bksv.com/products/transducers/vibration
  - www.bksv.com/products/shakers-and-exciters/modal-exciters
  - www.bksv.com/products/transducers/vibration/Vibration-transducers/impact-hammers
- YouTube videos on Structural Dynamics
  - Applications & solutions: (https://www.youtube.com/playlist?list=PLJiMDKQQaTpBFsa2kJ2KlMbgRzd_s7GYF)
  - Tutorials: (https://www.youtube.com/playlist?list=PLJiMDKQQaTpAxJ-Il200XRvV8RUmJHx3l)
- 4-week free trial software for structural dynamics
  - http://www.bksv.com/products/Analysis-software/structural-dynamics-software

Brüel & Kjær BEYOND MEASURE

www.bksv.com

HOTTINGER BRÜEL & KJÆR


Q&A

Thank You

niels-jorgen.jacobsen@hbkworld.com