Modal Analysis using Hammer or Shaker Excitation

CLASSICAL MODAL ANALYSIS, MOBILITY-BASED MODAL ANALYSIS, EXPERIMENTAL MODAL ANALYSIS, MODAL TESTING

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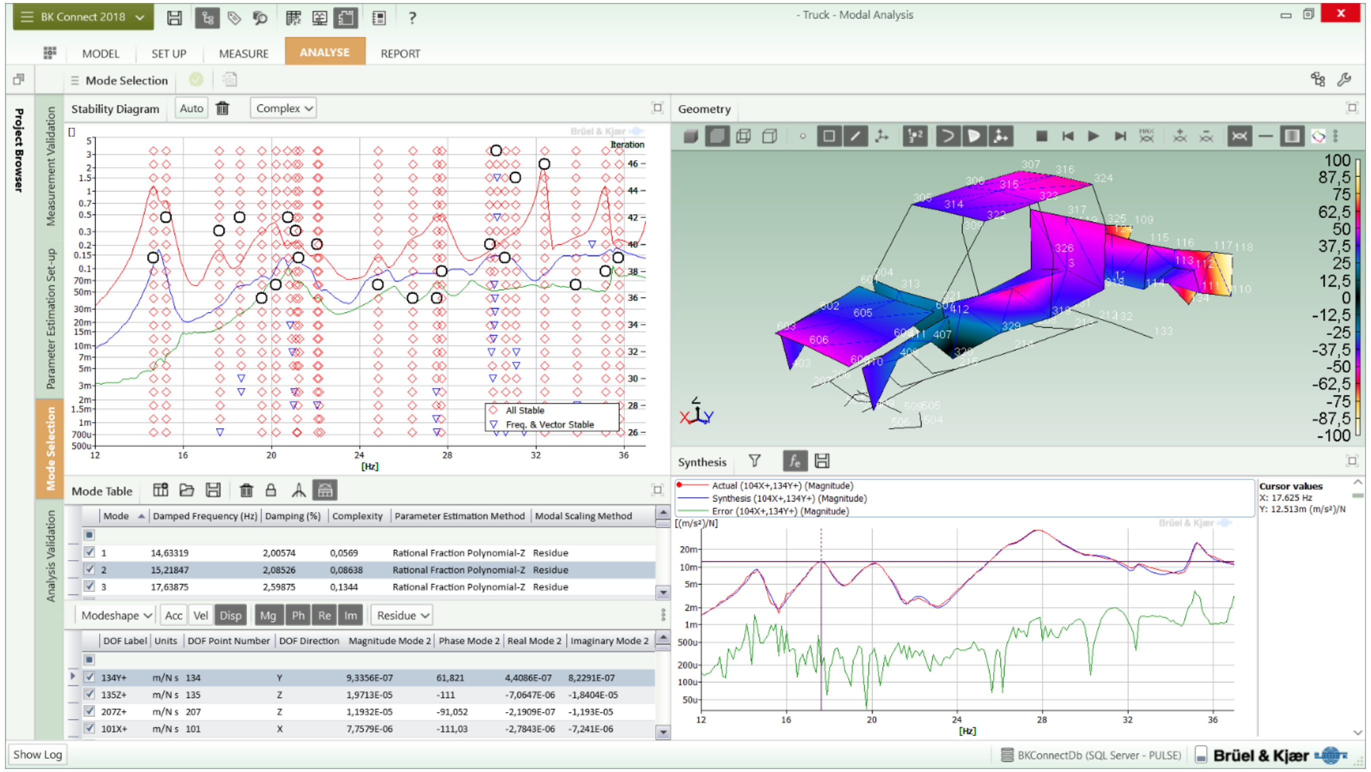
Classical Modal Analysis (Modal Testing)

To obtain a mathematical model of the dynamic properties of a structure by experimental means using hammer or shaker excitation *(modal testing)*

Controlled measurements
- Boundary conditions
- Excitation signals
- Environmental conditions

Excitation signals
- Sinusoidal (fixed, sweep, step)
- Random (normal, burst, pseudo …)
- Transient (impact)
- …

Results – Modes each consisting of three with modal parameters
- Natural frequencies
- Modal damping
- Mode shapes



Classical Modal Analysis answers the question:

What is the structure’s inherent dynamic properties?

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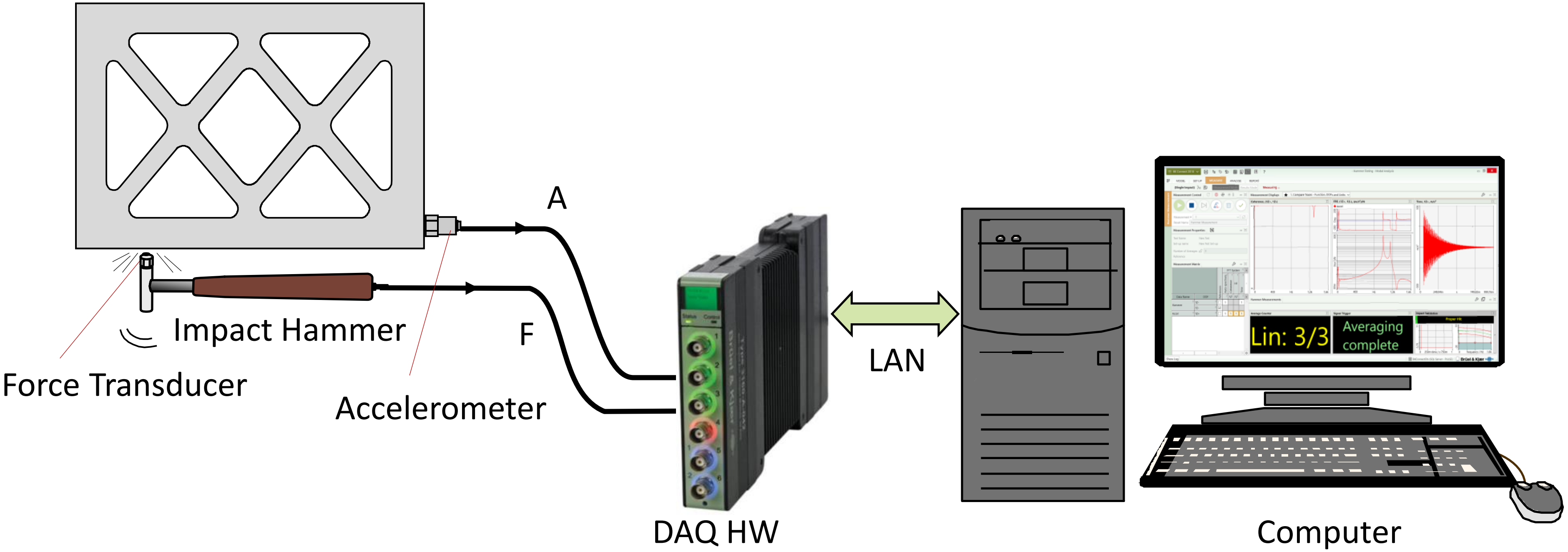
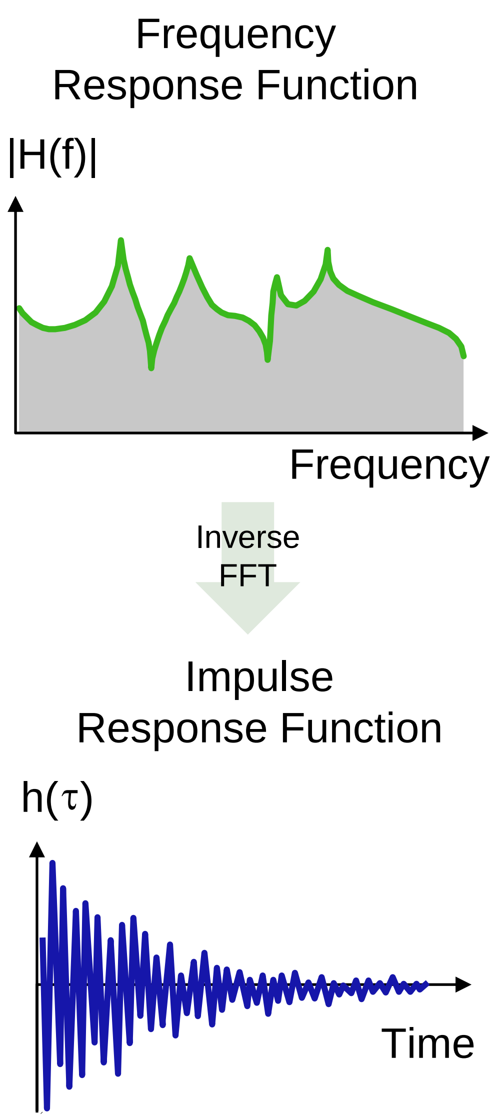
Mobility Measurements – Frequency Response Functions (FRFs) H(f)

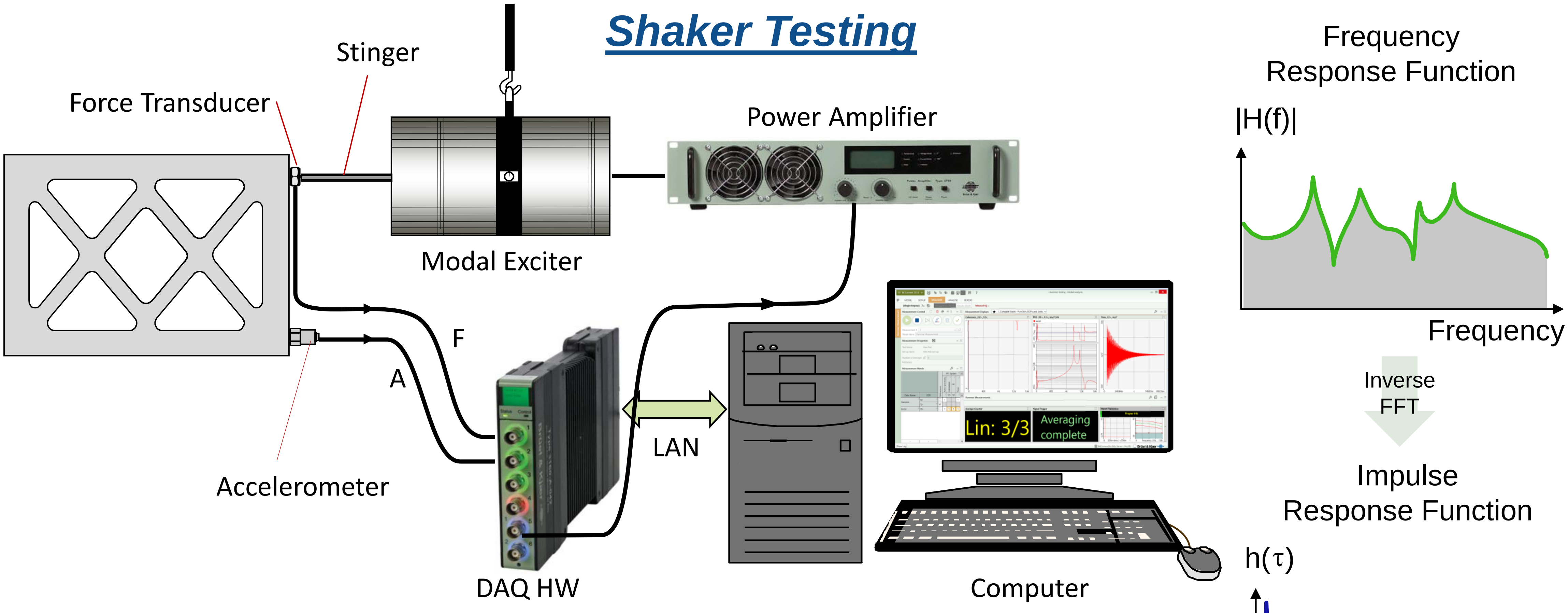
Hammer Testing

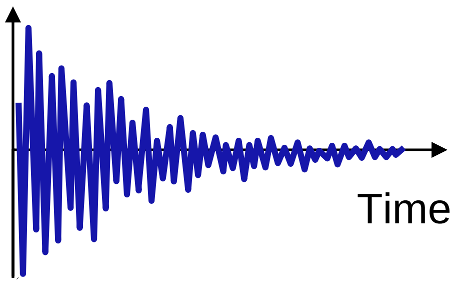




$$H(f) = \frac{\text{Output}}{\text{Input}} = \frac{\text{Vibration}}{\text{Force}} = \frac{\text{Response}}{\text{Excitation}}$$

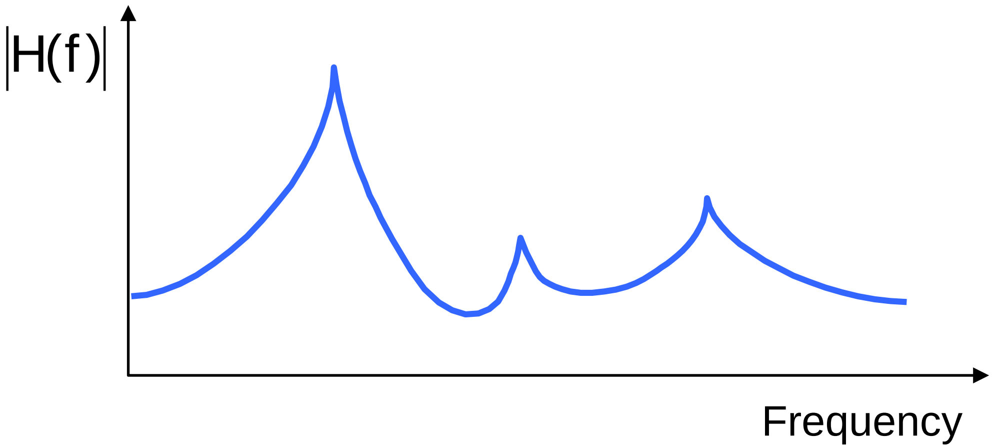
Mobility Measurements – Frequency Response Functions (FRFs) H(f)





$$H(f) = \frac{\text{Output}}{\text{Input}} = \frac{\text{Vibration}}{\text{Force}} = \frac{\text{Response}}{\text{Excitation}}$$

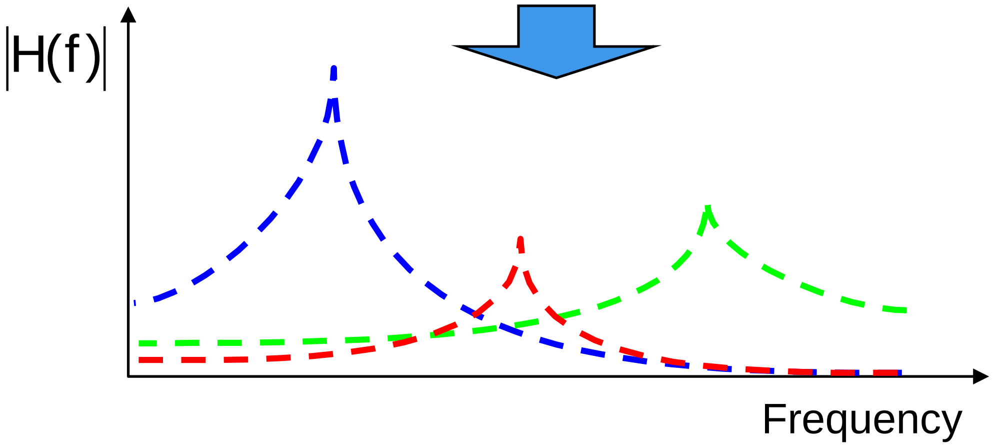
Mobility-based Modal Analysis (Classical Modal Analysis)



Measurement

MDOF model = Σ(SDOF models)

Curve-fitting
(Modal Parameter Extraction)



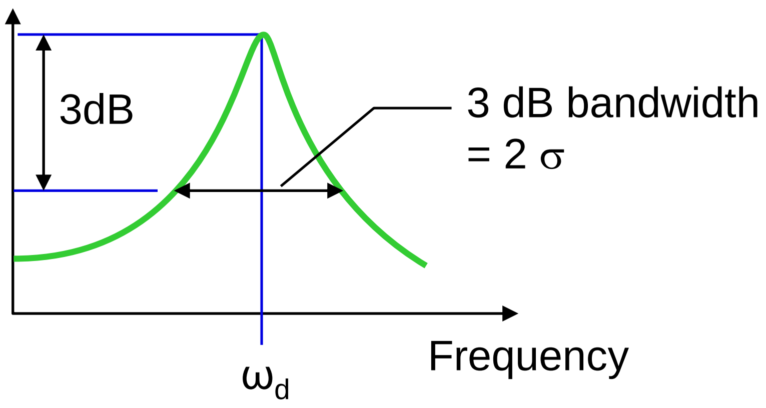
Modal Analysis

Global Parameters: Natural Frequency, Modal Damping; Local Parameter: Mode Shape

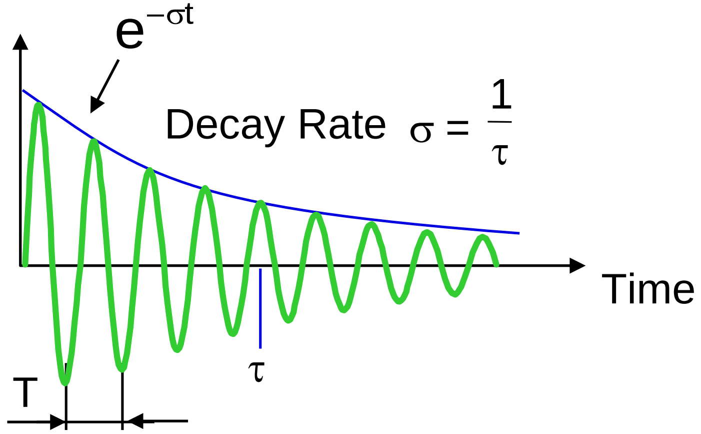
Modal model – Global parameters

Natural Frequency and Modal Damping
are characteristic for all DOF’s in the dynamic model (GLOBAL)

Frequency Domain



Time Domain



Natural Frequency: $f_d = \dfrac{\omega_d}{2\pi}$

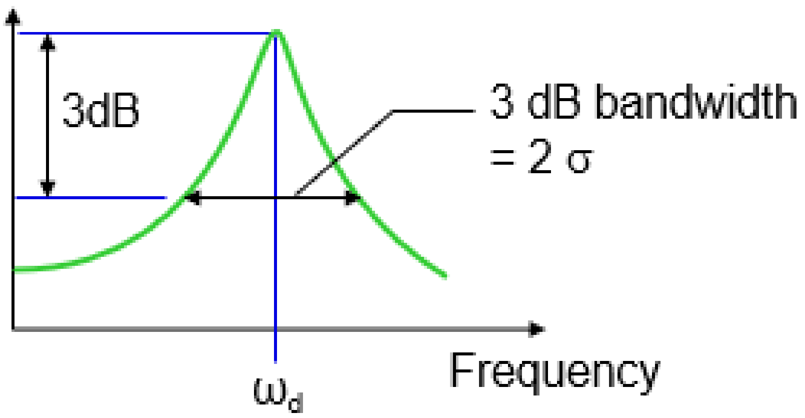
Damping Ratio: $\zeta = \dfrac{\sigma}{\omega_d}$

Natural Frequency: $f_d = \dfrac{1}{T}$

Damping Ratio: $\zeta = \dfrac{\sigma}{\omega_d}$

Modal model – Local parameter

A residue is characteristic for a single DOF in the dynamic model (LOCAL)



$$H(\omega) = \frac{R}{j\omega - p} + \frac{R^*}{j\omega - p^*}$$ where $p = -\sigma + j\omega_d$

$$H(j\omega) = \frac{R}{j\omega - (-\sigma + j\omega_d)} + \frac{R^*}{j\omega - (-\sigma - j\omega_d)}$$

At resonance: $|H(\omega_d)| \approx \frac{R}{\sigma}$

Residue: $R \approx |H(\omega_d)| \cdot \sigma$

Residue R: The “strength” of the mode

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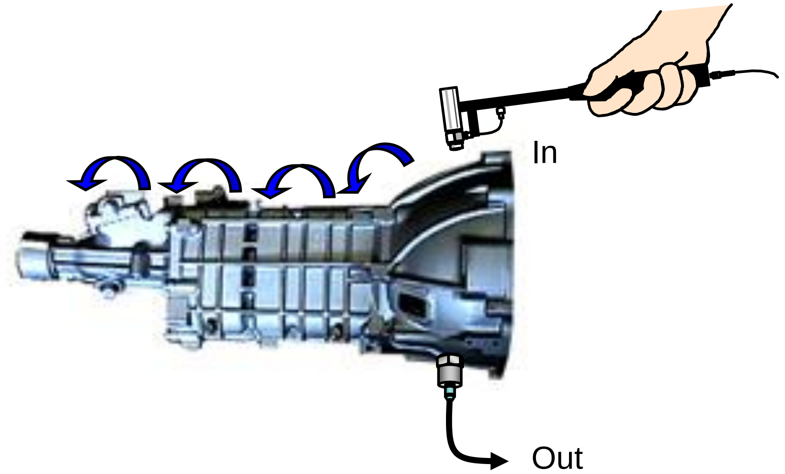
Excitation techniques

Hammer Excitation

Normally roving, but can be fixed

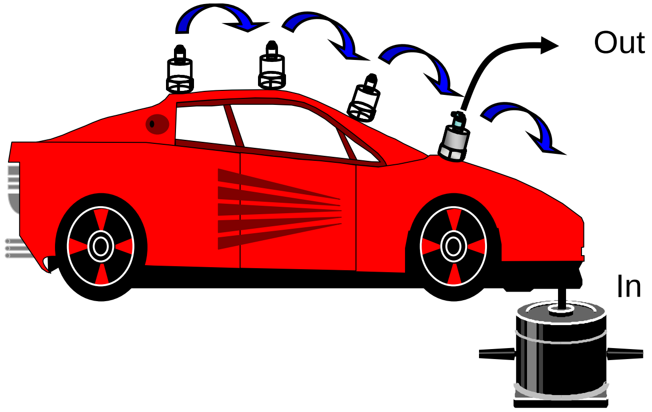


Roving => One Row

$$[H] = \begin{bmatrix} H_{11}\, H_{12} \cdots\cdots H_{1n} \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \end{bmatrix}$$

Shaker Excitation

Normally fixed, but can be roving



Fixed => One Column

$$[H] = \begin{bmatrix} H_{11} \cdots\cdots\cdots \\ H_{21} \cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ H_{n1} \cdots\cdots\cdots \end{bmatrix}$$

Multiple rows/columns *can be measured simultaneously by using* ***multiple accelerometers in hammer testing*** *or* ***multiple shakers (MIMO analysis)***

Required to find ***repeated roots/closely-coupled modes*** *and* ***local modes***

With MIMO analysis you get the excitation energy sufficiently distributed and can avoid non-linear behaviour

Modal analysis using hammer excitation

Conclusion

Well-suited for field work and trouble-shooting with main focus on ease-of-use

Advantages

- Fast
- No fixturing
- No variable mass loading
- Portable
- Relative inexpensive

Disadvantages

- High crest factor[1] might drive structure into non-linear behaviour
- High peak force needed for large structures might cause local damage
- Highly deterministic signal means no linear approximation

1) Crest factor = Peak/RMS.
A measure of how extreme peaks are in the signal

Modal analysis using shaker excitation

Advantages

- Broad range of excitation signals
  - Sine (single, step, swept, multi), Random (continuous, burst, periodic, pseudo), Pulse ...
- Optimization of numerous measurement parameters
  - Dynamic range, Signal-to-Noise ratio, Energy distribution, Crest factor, Linearity, Leakage ...
- Possibility of doing MIMO analysis
- Suitable for fixed large setups

Disadvantages

- Fixturing of structure, shakers and force transducers
- Potential dynamic loading from shakers and stinger
- Potential mechanical coupling between shakers (MIMO)
- Generally more expensive

Conclusion

Well-suited for laboratory work with main focus on flexibility and accuracy

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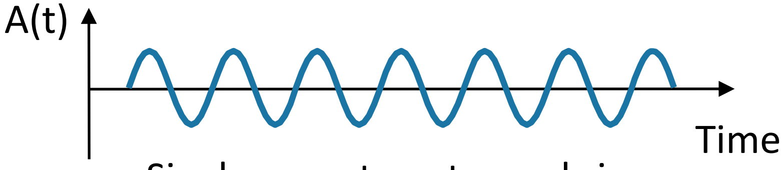
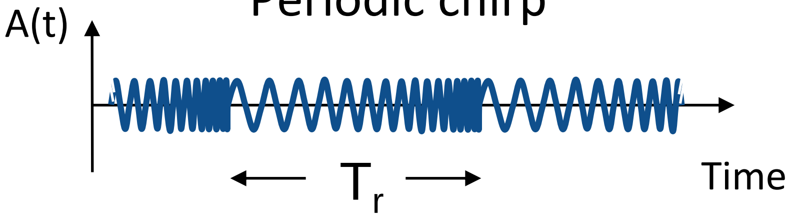
Examples of excitation signals

Sine

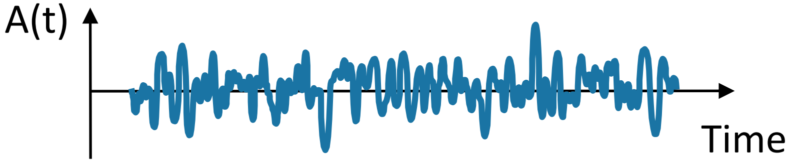


Single, swept or stepped sine

Periodic chirp

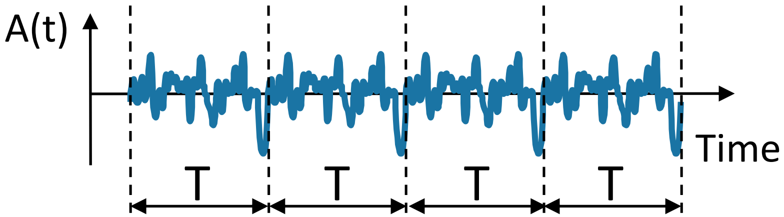


Continuous random

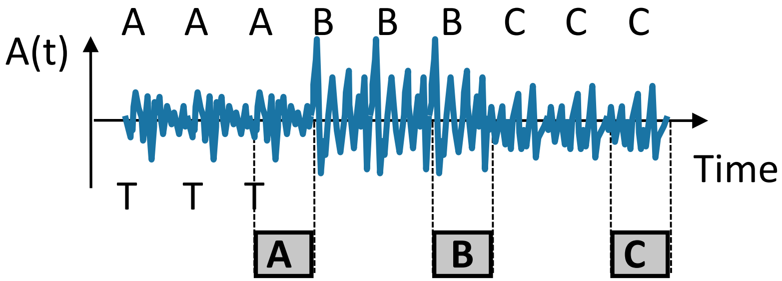


Burst random

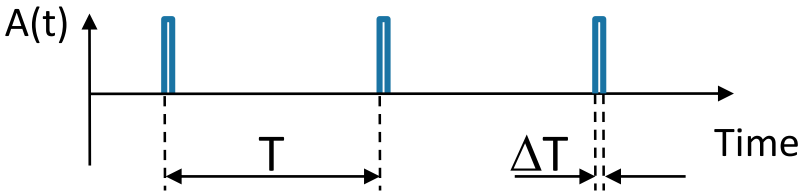
Pseudo random



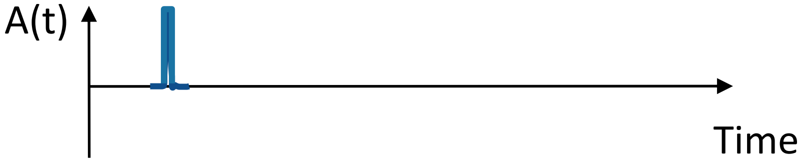
Periodic random



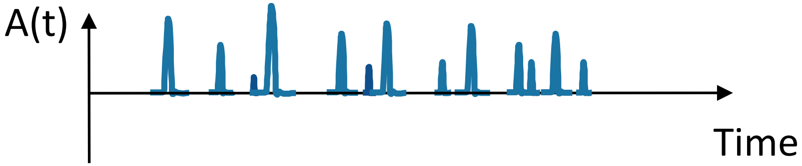
Periodic impulse



Single impact



Random impact



User-defined waveforms

Choice of excitation technique

The optimum excitation technique depends on

- Application
  - For example, are we taking about a complete modal analysis or just resonance testing?
- Non-linearities
  - Is the purpose to study the non-linearity or should the best linear approximation be obtained?
- Speed of test
  - Can we accept the compromise on accuracy, if test time is limited?
- Dynamic range and signal-to-noise ratio
  - Is it large enough to avoid overload and under-range situations?
- Leakage[1] (analysis)
  - How important is leakage for the structure being tested?
- Crest factor[2]
  - Can a high crest factor damage the system or excite non-linearities?
- Noise at input and/or output
  - Can the amount and effect of measurement noise be significantly reduced?
- Equipment available
  - Does the test system provide a sufficient range of excitation signals?
  - Can user-defined waveforms be generated and played-back?

1) In FFT analysis, leakage appears when the signal is not periodic wihin the used time blocks. Leakage causes the signal levels to be reduced and redistributed over a broad frequency range. Use of time windowing can mitigate the effects of leakage by smoothing a signal in the start and end of the time blocks before performing the FFT.

2) Crest factor = Peak/RMS.
A measure of how extreme peaks are in the signal

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Guidelines for choice of excitation technique

For study of non-linearities: →	Single, swept or stepped sine excitation
For slightly non-linear system: →	Random (continuous or burst) excitation
For perfectly linear system: →	Pseudo random excitation
For field measurements: →	Single impact excitation
For high-resolution field measurements: (Low frequency range) →	Random impact excitation

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Mode Indicator Functions (MIFs)

Tools for identification of **modes**, their **frequencies** and **density**

- MIFs are (normally) **real-value** frequency domain functions exhibiting a local **minima** or **maxima** at the **natural frequencies** of the modes
- Some MIFs only show a single function
  - Can only indicate separated modes
- Some MIFs show one function per reference
  - Can also indicate repeated roots and closely-coupled modes
- Available MIFs in BK Connect Modal Analysis
  - **Power** Mode Indicator Function (PMIF)
  - **Normal** Mode Indicator Function (NMIF)
  - **Complex** Mode Indicator Function (CMIF)
  - **Multivariate** Mode Indicator Function (MMIF)

	Single Function	Multiple Functions	Mode Identification
Response-based	***PMIF***	***CMIF***	Maxima (peaks)
Normalized	***NMIF***	***MMIF***	Minima (valleys)

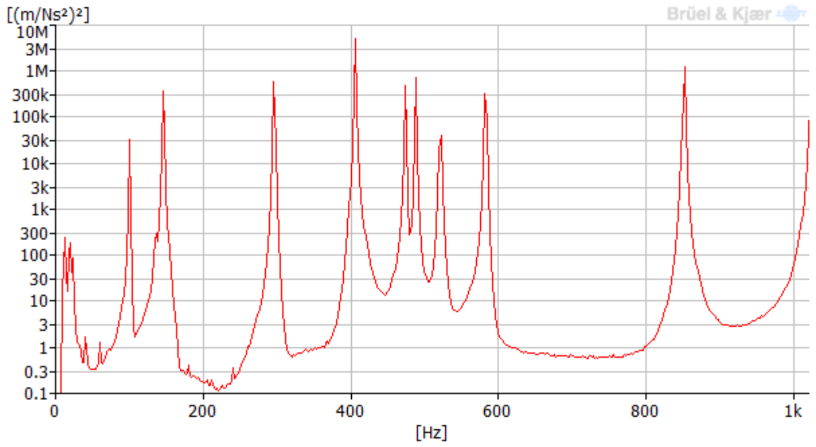
- Frequency Response Functions (FRFs) can be used as **simple MIFs**

MIFs – Single Functions

- **PMIF (Power MIF)**
  - Sum of Squared FRFs values (Real, Imag, Mag)

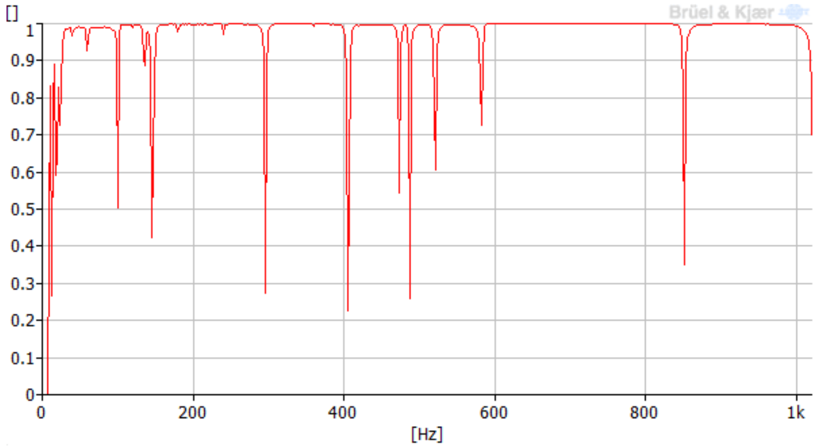
$$\sum(H_{\mathrm{Re}}^2) \quad \sum(H_{\mathrm{Im}}^2) \quad \sum(|H|^2)$$

  - For better selectivity use Imag. Part for Acceleration FRFs (A/F)



- **NMIF (Normal MIF)**
  - Minima found from zero-crossing of the real part at resonances
  - Scaled from 0 to 1

$$\frac{\sum|\mathrm{Re}(H)||H|}{\sum(|H|^2)}$$



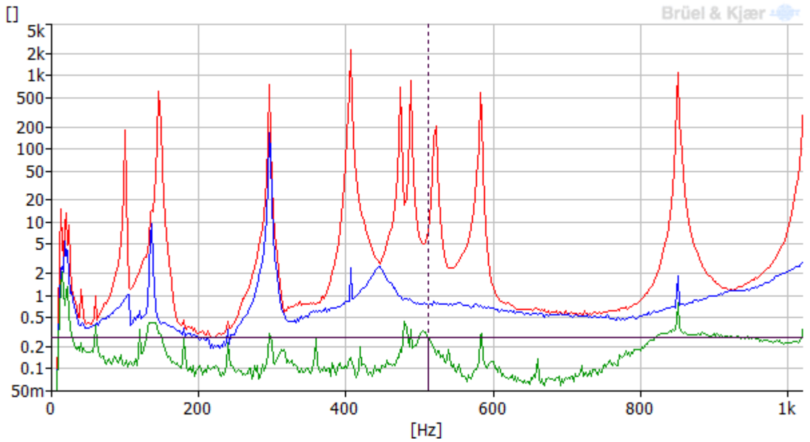
Sum over one or more references (use Function Selector)

MIFs – Multiple Functions

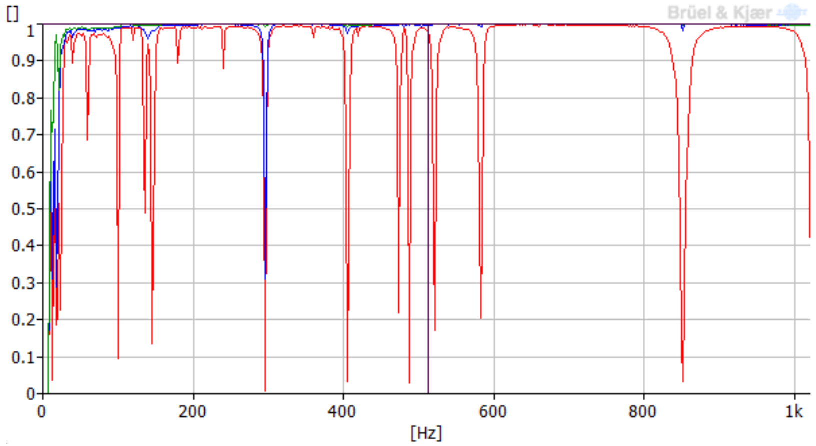
- **CMIF (Complex MIF)**
  - Plots the singular values from a Singular Value Decomposition (SVD) of the FRF matrix

$$[H(\omega)] = [U(\omega)]\left[\sum(\omega)\right][V(\omega)]^H$$

  - One function for each reference
  - For better selectivity use Imag. Part for Acceleration FRFs (A/F)



- **MMIF (Multivariate MIF)**
  - Minimization of real part of response compared to total response (Corresponds to identification of normal modes)
  - Scaled from 0 to 1
  - One function for each reference



Select number of references to be used from the Function Selector

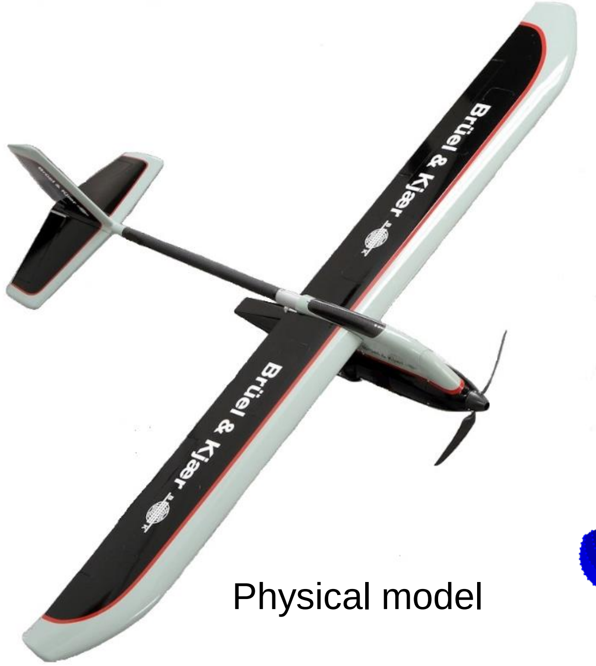
Example - Demo of MIFs

Aircraft:

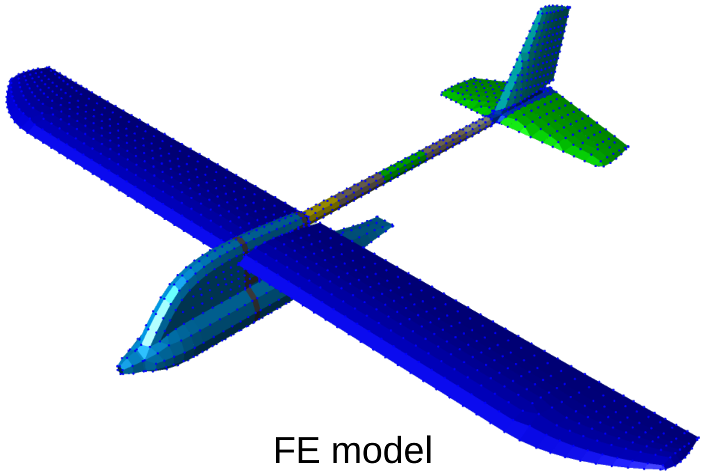
- Wingspan: 3m
- Length: 2m
- Weight: 17 kg (37 lb)
- Composite material
- Assembled by clicking 7 parts together

Test configuration:

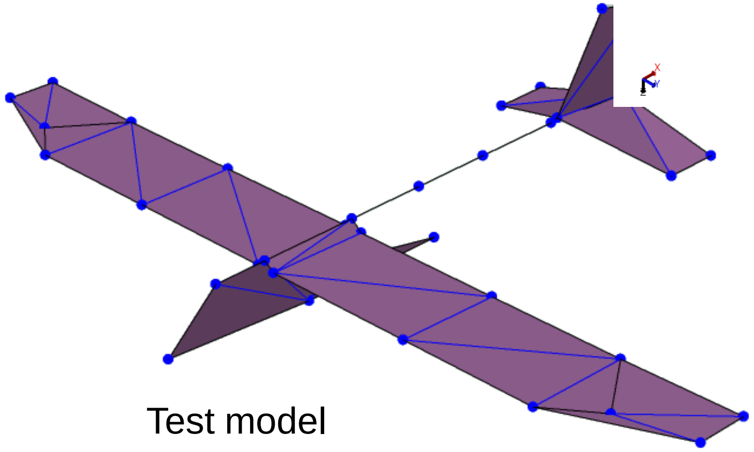
- Geometry
  - – 36 nodes; 32 elements
- 2 shakers close to the wing tips
- 17 uniaxial and 26 triaxial accelerometers
- 95 DOFs and 190 FRFs
- Freq. range: 0-50 Hz



Physical model



FE model



Test model

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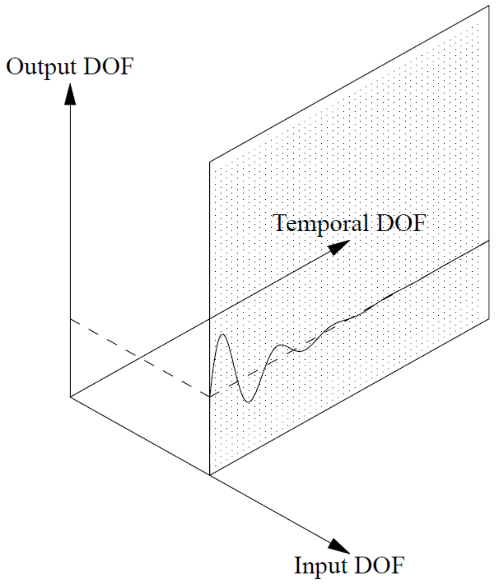
Modal parameter estimation – Curve-fitting

Identification of a **system model** described by the **modal parameters** (*natural frequencies*, *damping ratios* and *mode shapes*) obtained from **measurements**

Curve-fitting is a large data reduction process. Consequently, no single <u>ideal</u> curve-fitter exists!

- **Over-determined problem** (more data than independent equations)
- The challenge is to properly reduce the data (spatial and temporal information) to a certain subspace containing one (or preferable more) planes in the **Characteristic Space**



Characteristic Space

The particular subspace measured and the weighting of the data within the subspace are the main differences between the various modal parameter estimation techniques developed!

Classification of curve-fitters (1..3)

Curve-fitters can be classified according to the following parameters

- **Single Reference**
  - Can only solve for **one row/column** in the FRF matrix
  - For example, fixed single shaker test or roving hammer test with single response accelerometer
  - Only for structures with separated modes

- **Poly-reference**
  - Can solve for **multiple rows/columns** in the FRF matrix
  - For example, fixed multiple shaker test or roving hammer test with multiple response accelerometers
  - Also for structures with repeated roots and closely-spaced modes

- **Frequency Domain**
  - Based on Frequency Response Functions (FRFs)

- **Time Domain**
  - Based on Impulse Response Functions (IRFs)

Classification of curve-fitters (2..3)

- **Single Degree-of-Freedom (SDOF)**
  - One mode estimated at a time
  - Only for structures lightly to medium damped and with separated modes
- **Multiple Degree-of-Freedom (MDOF)**
  - Multiple modes estimated simultaneously (using a band cursor)
  - Also for heavily damped structures and structures with repeated roots/closely-spaced modes
- **Global Solve**
  - For curve-fitting of **consistent FRF data**
  - Pole location (frequency and damping) estimated in a least squares sense from **all the measurements** in the data set
  - The **global pole location** values are then used to fit the residue (~ mode shape) for each individual measurement
- **Local Solve**
  - For curve-fitting of **inconsistent FRF data**, for example, due to variable mass loading
  - Pole locations are estimated from **one** or a **few measurements** and then averaged across all local fits to provide a single estimate for each mode
  - Residues fitted for each **individual pole position**

Classification of curve-fitters (3..3)

General formulation using Matrix Coefficient Polynomial model

$$H(s) = \frac{\beta_n (s)^n + \beta_{n-1} (s)^{n-1} + \ldots + \beta_0}{\alpha_m (s)^m + \alpha_{m-1} (s)^{m-1} + \ldots + \alpha_0}$$

Natural Frequencies and **Damping Ratios** (poles) from

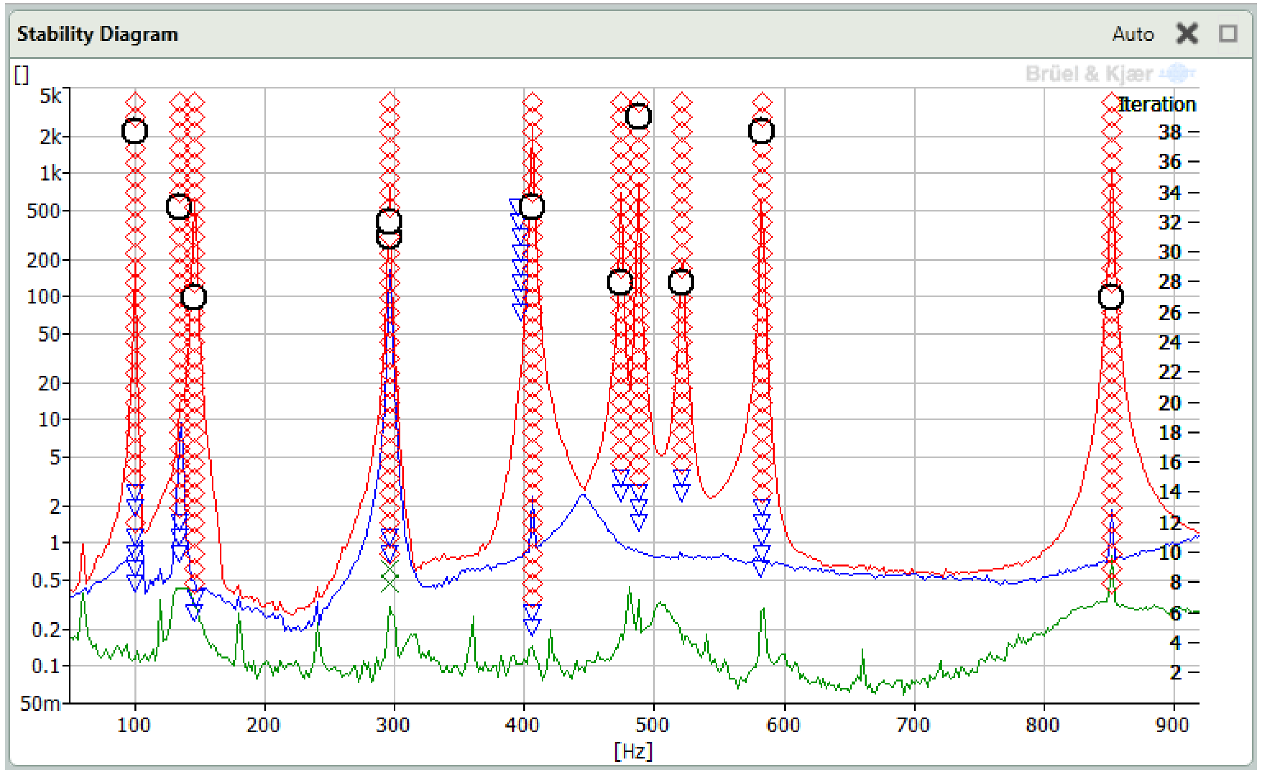
$$\left| [\alpha_m] s^m + [\alpha_{m-1}] s^{m-1} + \ldots + [\alpha_0] \right| = 0$$ Frequency Domain

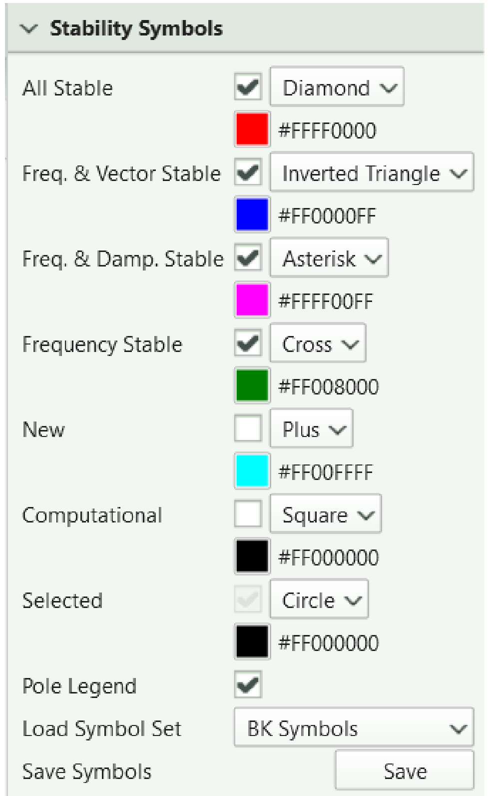
$$\left| [\alpha_m] z^m + [\alpha_{m-1}] z^{m-1} + \ldots + [\alpha_0] \right| = 0$$ Time Domain

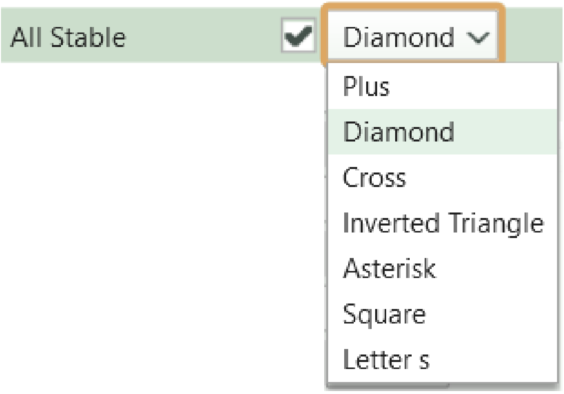
Number of poles given by order of m and size of [α] (spacial dimension)
High Order methods (using order of polynomial, m)
Low Order methods (using spacial dimension, [α] – virtual DOFs)

Stability diagram

- Plots stability symbols for iteration (model order) vs. frequency. Mode indicator function as background
- Stable modes appear as vertical lines
- Modes selected manually by double-clicking on symbols or automatically by pushing a single button







Curve-fitters in BK Connect Modal Analysis

Single Degree-of-Freedom (SDOF) Curve-fitters

- Quadrature (peak-picking); Least Squares Global Partial Fraction

Multiple Degree-of-Freedom (MDOF) and Global Solve Polyreference Curve-fitters

Order	Domain	
	Time	Frequency
Low	*Eigensystem Realization (ERA)*	*Polyreference Frequency (PFD)*
High	*Polyreference Time (PTD)*	*Rational Fraction Polynomial-Z (RFP-Z)*

*Our implementation adds **Enhanced Mode Solution** for very clear stability diagrams for the **PTD** and **RFP-Z** curve-fitters*

Automated Mode Selection available for all MDOF curve-fitters!
Local solve available for the PTD and RFP-Z curve-fitters!

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Analysis validation tools

Tools for Validation of the obtained Modal Analysis Results

- **Synthesis**
  - Comparison of synthesised to measured FRFs (or MIFs)
  - Investigation of effects of removing modes from the mode table
- **Mode Shape Animation**
  - Single, Side-by-Side, Top-Bottom, Overlaid, Difference …
  - Wireframe, Surfaces, Contour, Deformed/Undeformed, Max. Deformation ...
- **Complexity Plot and Mode Normalization**
  - Validation of the complexity of modes
  - Normalization of the modes using an ”angle indicator”
- **AutoMAC and CrossMAC** (MAC: Modal Assurance Criterion)
  - Comparison of the linearity of mode shape pairs for one set (AutoMAC) or two sets (CrossMAC) of mode shapes
  - Normalized between 0 and 1 (1: exact same shape; 0: completely different shapes)
- **CoMAC** (CoMAC: Coordinate MAC)
  - Comparison of two sets of mode shapes on a *DOF basis*
  - The method seeks to identify the parts of the structure that are responsible for low degrees of correlation (shown on geometry display)

Contents

Further Information - Brüel & Kjær

- **Knowledge Centre** (www.bksv.com/Knowledge-center)
  - Case Studies, Application Notes, Technical Reviews, Conference Papers, Primers and Handbooks

- **Training** (www.bksv.com/Training)
  - Courses, Webinars, Customized Training, Video Tutorials
    - Includes 3-day course on Classical Modal Analysis and 2-day course on Operational Modal Analysis

- **Structural Dynamics - Product and Application pages**
  - Operational Modal Analysis, Classical Modal Analysis, ODS Analysis, Structural Health Monitoring, Test-FEA Integration …
  - Software, hardware, transducers, modal exciters, impact hammers …
  - www.bksv.com/Applications/product-vibration/structural-dynamics
  - www.bksv.com/products/Analysis-software/structural-dynamics-software
  - www.bksv.com/products/data-acquisition-systems-and-hardware/LAN-XI-data-acquisition-hardware
  - www.bksv.com/products/transducers/vibration
  - www.bksv.com/products/shakers-and-exciters/modal-exciters
  - www.bksv.com/products/transducers/vibration/Vibration-transducers/impact-hammers

- **YouTube videos on Structural Dynamics**
  - Applications & solutions: (https://www.youtube.com/playlist?list=PLJiMDKQQaTpBFsa2kJ2KlMbgRzd_s7GYF)
  - Tutorials: (https://www.youtube.com/playlist?list=PLJiMDKQQaTpAxJ-Il200XRvV8RUmJHx3l)

- **4-week free trial software for structural dynamics**
  - http://www.bksv.com/products/Analysis-software/structural-dynamics-software

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Previous 1-hour webinars on structural dynamics

February 19, 2020

- Introduction to Structural Dynamics – Measurements and Analysis
  - Recording: https://www.hbm.com/en/8722/introduction-to-structural-dynamics-measurement-and-analysis/

February 20, 2020

- Operating Deflection Shapes Analysis – Determination of Vibration Patterns under Operating Conditions
  - Recording: https://www.hbm.com/en/8740/ods-vibration-patterns-under-operating-conditions/

November 20, 2019

- Operational Modal Analysis – Modal Parameter Identification under Operating Conditions
  - Recording: https://www.hbm.com/en/8507/webinar-operational-modal-analysis-modal-parameter-identification-under-operating-conditions/


Q&A

Thank You

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